A Model of Job and Worker Flows*

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Abstract

We develop a model of gross job and worker flows and use it to study how the wages, permanent incomes and employment status of individual workers evolve over time and how they are affected by aggregate labor market conditions. Our model helps explain various other features of labor markets, such as the size and persistence of the changes in income that workers experience due to displacements or job-to-job transitions, the length of job tenures and unemployment duration, and the amount of worker turnover in excess of job reallocation. We also examine the effects that labor market institutions and public policy have on the gross flows, as well as on the resulting wage distribution, employment and aggregate output in the equilibrium. From a theoretical point of view, we study the extent to which the competitive equilibrium achieves an efficient allocation of resources.

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*This draft is preliminary and incomplete. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

Recent empirical and theoretical studies on gross job and worker flows have changed the way we think about the labor market. We now know that market economies exhibit high rates of reallocation of employment across establishments as well as high rates of worker turnover from one job to another and between employment and unemployment. We now view the number of employed or unemployed workers as resulting from a large and continual reallocation process, and we analyze how changes in public policy and the economic environment affect this process. The study of the gross flows provides valuable hints on how the labor market carries out this continual reallocation of resources, and at the same time raises many interesting questions: To what extent are market economies able to perform this reallocation process efficiently? How is this process affected by labor market policies? What determines the amount of worker turnover in excess of job reallocation?

The empirical literature distinguishes between measures of job flows and worker flows. In a series of influential studies using U.S. manufacturing census data, Davis and Haltiwanger (1992, 1999) and Davis, Haltiwanger and Schuh (1996) measure gross *job creation* (JC_t) as the sum of employment gains over all plants that expand or start up between dates $t-1$ and t; gross *job destruction* (JD_t) as the sum of employment losses over all plants that contract or shut down; and gross *job reallocation* as the sum of gross job creation and destruction ($JR_t = JC_t + JD_t$). By showing that gross job creation and destruction are both large irrespective of whether aggregate employment grows or declines, their work highlights the role of heterogeneous forces that cause employment to expand in some plants and contract in others. Behind these large job flows, however, are even larger worker flows.

Estimates of worker flows are based on establishment or worker surveys and measure the movements of workers across establishments and labor-market states. Empirical studies that draw on establishment data often define *worker turnover* at establishment i (WT_{it}) as the sum of the number of accessions (new hires) and separations (quits and displacements) between dates

$t-1$ and t, and aggregate worker turnover (WT_t) as the sum of worker turnover over establishments. The number of workers who quit, get displaced, and get hired by each establishment is at least as large as (and often significantly larger than) the net change of employment at that establishment. Burgess, Lane and Stevens (2000), for example, refer to the difference between worker turnover and the net employment change as "churning" ($C_{it} = WT_{it} - |e_{it} - e_{it-1}|$, where e_{it} is employment in establishment i at the end of period t). This notion of churning measures the number of worker transitions in excess of the minimum level needed to achieve the actual change in employment. Summing over establishments delivers an aggregate measure of churning: $C_t = WT_t - JR_t$.

Alternatively, using data from worker surveys we can define *worker reallocation* (WR_t) as the number of workers who change employment states (i.e., who change place of employment, find or lose a job, or enter or exit the labor force) between dates $t-1$ and t. Worker turnover measures the number of labor market transitions, while worker reallocation counts the number of workers who participate in transitions. A worker who moves from one establishment to another increases the worker reallocation count by one and the aggregate worker turnover count by two; hence, aggregate worker turnover is larger than worker reallocation by the number of job-to-job transitions.[1]

Drawing from different data sources for job and worker flows, Davis and Haltiwanger (1992) estimate that job creation and destruction account for no less than one-third and no more than one-half of quarterly worker turnover in the U.S. manufacturing sector. New evidence from data sets that incorporate information on the number of accessions and separations at the establishment level indicates that for most establishments, for most of the time, worker turnover is much larger than job reallocation. For example, Burgess, Lane and Stevens (2000) use quarterly data from all private sector establishments in the state of Maryland and find that churning flows account for 70% of worker turnover in nonmanufacturing and about 62% in

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[1] This is the case provided both the worker-side and establishment-side data sets cover the entire economy and provided no accessions or separations are reversed within the sample period; else $WT_t - WR_t$ would be an upper bound for the number of job-to-job transitions.

manufacturing (job reallocation accounts for the rest). Similarly, based on data derived from the unemployment insurance systems of eight U.S. states, Anderson and Meyer (1994) report that gross job reallocation accounts for only 24% of quarterly worker turnover in manufacturing. Drawing from a data set covering the universe of Danish manufacturing plants, Albæk and Sørensen (1998) report a ratio of quarterly job reallocation to worker turnover of .42 and find that replacement hiring (defined as the sum of accessions minus job creation) is on average 16.5% of manufacturing employment.[2] Hamermesh, Hassink and van Ours (1994) find that job reallocation is only one-third of worker turnover in a random sample of establishments in the Netherlands. They also find that most mobility is into and out of existing jobs, not to new or from destroyed jobs; that a large fraction of all hires (separations) take place at firms where employment is declining (expanding); and that simultaneous hiring is mostly due to unobservable heterogeneity in the workforce.

From an aggregate perspective, the amount of worker reallocation in excess of job reallocation depends not only on the amount of simultaneous hiring and firing that takes place at the establishment level (as measured by C_{it}), but also on the extent to which job-to-job transitions are a common mechanism through which the market achieves the reallocation of workers. In this respect, recent studies find that job-to-job flows are large: Fallick and Fleischman (2001) estimate that in the United States in 1999, on average 4 million workers changed employers from one month to the next (about 2.7% of employment), more than twice the number who transited from employment to unemployment.

The fact that worker flows exceed job flows at the establishment level is evidence of heterogeneity over and above the cross-establishment heterogeneity that can be inferred from the size of the job flows alone. And the large employment-to-employment flows indicate that job-to-job

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[2] Albæk and Sørensen report interesting cross-establishment observations as well. For example, they report that 62% of all separations are accounted for by plants with employment growth rates in the interval $(-0.3, 0.1]$ and that plants with employment growth rates in the interval $(-0.1, 0.3]$ account for 56% of all hires. Burgess, Lane and Stevens (2000) also present some establishment-level cross-sectional evidence, such as that most of the employers in their data set have churning rates above 50%. (See their Figure 1 on page 483, which reports the distribution of C_{it}/W_{it}.)

transitions play a prominent role in the reallocation process. All this suggests that, in order to understand the process that reallocates workers and employment positions in actual labor markets, we must study the nature of job-to-job transitions and the implications of heterogeneity at the level of the employer-worker match.

In this paper, we develop an equilibrium search model that distinguishes between gross job and worker flows, incorporates job-to-job transitions, and exhibits instances of replacement hiring.[3] We use the model to study how the employment status and wages of individual workers evolve over time and how they are affected by aggregate labor market conditions. We also examine the effects that labor market institutions and public policy have on the gross job and worker flows, as well as on the resulting wage distribution, employment and aggregate output in the equilibrium. In addition, our model helps explain various other features of labor markets. For example, why do displaced workers tend to experience a significant and persistent fall in incomes? Why do workers stay unemployed when on-the-job search is at least as effective as off-the-job search? Why are good jobs not only better paid, but often also more stable?

The rest of the paper is organized as follows. Section 2 lays out the environment. Section 3 defines and characterizes the salient features of the equilibrium. For a special case, Section 4 provides a fuller characterization of the equilibrium set and discusses the main properties of the allocations. Section 5 incorporates employment protection policies. Section 6 extends the model to allow for free entry of employers. Section 7 discusses some of the related theoretical literature on labor market matching models with on-the-job search. Section 8 concludes. The Appendix contains proofs and explains some properties of the bargaining procedure we propose.

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[3] Job and worker reallocation are one and the same by construction in the workhorse of much of the recent macro-labor literature, the matching model of Mortensen and Pissarides (1994) or Pissarides (2000). And there is no room for replacement hiring in the influential on-the-job search model of Burdett and Mortensen (1998).

2 The Model

Time is continuous and the horizon is infinite. The economy is populated by a continuum of fixed and equal numbers of workers and employers.[4] We normalize the size of each population to unity. Workers and employers are infinitely-lived and risk-neutral. They discount future utility at rate $r > 0$, and are *ex ante* homogeneous in tastes and technology.

A worker meets a randomly chosen employer according to a Poisson process with arrival rate α. An employer meets a random worker according to the same process. Upon meeting, the employer-worker pair randomly draws a production opportunity of productivity y, which represents the flow net output each agent will produce while matched. (Thus the pair produces $2y$.) The random variable y takes one of N distinct values: $y_1, y_2, \ldots, y_N$, where $0 < y_1 < y_2 < \ldots < y_N$, and $y = y_i$ with probability π_i for $i = 1, ..., N$, and $\sum_{i=1}^{N} \pi_i = 1$. For now, we assume y remains constant for the duration of the match.[5]

Matched and unmatched agents meet potential partners at the same rate, so when an employer and a worker meet and draw a productive opportunity each of them may or may not already be matched with an old production partner. Each worker and employer can form at most one productive partnership simultaneously. The realization of the random variable y that an employer and worker draw when they first meet is observed without delay by them as well as by their current partners. In fact, the productivity of the new potential match as well as the productivities of the existing matches are public information to all the agents involved, i.e. the worker and the employer who draw the new productivity and their existing partners if they have any. On the other hand, each agent's history is private information, except for what is revealed by the current production match.

When a worker and an employer meet and find a new productive opportunity, the pair and

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[4] Although our main interest here is in the labor market, our model is applicable to any other setting where bilateral partnerships are relevant, such as the interactions between spouses, or between a tenant and a landlord, or between a supplier and the buyer of a customized product.

[5] In this basic setup, employers and workers are distinguised by type only in that each match requires exactly one partner of each type. Below we analyze extensions where employers and workers are different in a variety of ways.

their old partners (if they have any) determine whether or not the new match is formed (and consequently whether or not the existing matches are destroyed) as well as the once-and-for-all side payments that each party pays or receives, through a bargaining protocol which we will describe shortly. Utility is assumed to be transferable among all the agents involved in a meeting. There is no outside court to enforce any formal contract, so that any effective contract must be self-enforcing among the parties involved. If the parties who made contact decide to form a new partnership, they leave their existing partners who then become unmatched. In addition to these endogenous terminations, we assume any match is subject to exogenous separation according to a Poisson process with arrival rate δ.

We use n_{it} to denote the measure of matches of productivity y_i and n_{0t} to denote the measure of unmatched employers or workers at date t. Let τ_{ijt}^k be the probability that a worker with current productivity y_i and an employer with current productivity y_j form a new match of productivity y_k, given that they draw an opportunity to produce y_k at time t. (Hereafter, we will suppress the time subindex when no confusion arises.) The measure of workers in each state evolves according to:

$$
\begin{aligned}
\dot{n}_i &= \alpha \pi_i \sum_{j=0}^{N} \sum_{k=0}^{N} n_j n_k \tau_{jk}^i - \alpha n_i \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k (\tau_{ij}^k + \tau_{ji}^k) - \delta n_i && (1) \\
\dot{n}_0 &= \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} n_i n_j \pi_k \tau_{ij}^k + \delta \sum_{j=1}^{N} n_j - \alpha n_0^2 \sum_{k=1}^{N} \pi_k \tau_{00}^k. && (2)
\end{aligned}
$$

The first term on the right hand side of (1) is the flow of new matches of productivity y_i created by all types workers and employers. The second term is the total flow of matches with productivity y_i destroyed endogenously when the worker or the employer leaves to form a new match. The last term is the flow of matches dissolved exogenously. On the right hand side of equation (2), the first term is the flow of workers who become unmatched when their employers decide to break the current match to form a new match with another worker. The second term is the flow of workers who become unmatched due to the exogenous dissolution of matches. The third term is the flow of new matches created by unmatched workers and employers. (The creation of a new match involving an unmatched agent and a matched agent does not affect the aggregate number of unmatched agents, since one previously unmatched agent becomes matched, while one previously matched agent looses the partner to become unmatched.)

Before describing the competitive matching equilibrium with bargaining, we solve the social planner's problem. The planner chooses $\tau_{ij}^{k} \in [0,1]$ to maximize the discounted value of aggregate output:

$$\int_0^\infty e^{-rt} \sum\nolimits_{i=1}^{N} 2y_i n_i dt$$

subject to the flow constraints (1) and (2), and initial conditions for n_0 and n_i for $i = 1, ..., N$. Letting λ_i be the shadow price of a match with productivity y_i at date t, the Hamiltonian is

$$H = \sum_{i=1}^{N} 2y_i n_i - \delta \sum_{i=1}^{N} (\lambda_i - \lambda_0) n_i + \alpha \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{N} n_i n_j \pi_k \tau_{ij}^{k} (\lambda_k + \lambda_0 - \lambda_i - \lambda_j) .$$

The optimality conditions are:

$$\tau_{ij}^{k} \begin{cases} = 1 & \text{if } \lambda_k + \lambda_0 > \lambda_i + \lambda_j \\ \in [0,1] & \text{if } \lambda_k + \lambda_0 = \lambda_i + \lambda_j \\ = 0 & \text{if } \lambda_k + \lambda_0 < \lambda_i + \lambda_j \end{cases} \tag{3}$$

together with the Euler equations,

$$\begin{aligned} r\lambda_i - \dot{\lambda}_i &= 2y_i - \delta(\lambda_i - \lambda_0) + \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k (\tau_{ij}^{k} + \tau_{ji}^{k}) (\lambda_k + \lambda_0 - \lambda_i - \lambda_j) , \\ r\lambda_0 - \dot{\lambda}_0 &= \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k (\tau_{0j}^{k} + \tau_{j0}^{k}) (\lambda_k - \lambda_j) , \end{aligned}$$

and (1) and (2), for a given initial condition for n_0 and n_i at date 0. According to (3), to achieve the optimal allocation the planner specifies that a type i worker and type j employer should form a new match of productivity y_k for sure, if and only if the sum of the shadow prices of the new match and the unmatched worker and employee (which the new match would generate) exceeds the sum of the shadow prices of the existing matches of productivity y_i and y_j. From (3) we also learn that $\tau_{ij}^{k} = \tau_{ji}^{k}$, possibly except for the case of randomized strategies. Intuitively, there is no inherent asymmetry between a worker and an employer, so the planner treats them symmetrically in the optimal allocation. These observations allow us to summarize the first order necessary conditions as:

$$r\lambda_i - \dot{\lambda}_i = 2y_i - \delta\left(\lambda_i - \lambda_0\right) + 2\alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{0 \le \tau_{ij}^k \le 1} \tau_{ij}^k \left(\lambda_k + \lambda_0 - \lambda_i - \lambda_j\right) \tag{4}$$

$$r\lambda_0 - \dot{\lambda}_0 = 2\alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{0 \le \tau_{0j}^k \le 1} \tau_{0j}^k \left(\lambda_k - \lambda_j\right). \tag{5}$$

3 Competitive Matching Equilibrium

In this section we characterize the competitive matching equilibrium with the following bargaining procedure. When an agent draws an opportunity to produce with a new partner, with probability a half, she makes take-it-or-leave-it offers to her new potential partner and her old partner (if she has one) about production and side payments. She can rank these two offers, by making her offer to the old partner contingent on her offer to the new potential partner being rejected. With another probability half, her new potential partner and her old partner (if she has one) simultaneously make take-it-or-leave-it offers to her. After these offers are made, the recipient of the offers chooses which one to accept. We also specify that matched agents split the surplus symmetrically as long as neither agent encounters a production opportunity with another potential partner.[6]

Because a worker and an employer who form a match are inherently symmetric, hereafter we restrict our attention to symmetric equilibria in which workers and employers are treated symmetrically and are distinguished only by the productivity of their current match (or unmatched state). We will refer to a match of productivity y_i as a "type i match", and call a worker or an employer in a type i match a "type i agent". Let V_i be the value of expected discounted utility of a type i agent (either a worker or employer), and let V_0 be the value of an unmatched agent. Let X_{ij}^k be the value that a type i agent offers to a type j agent in order to form (or preserve) a match of productivity y_k. Specifically, X_{ij}^k includes the value of the new match plus the net side

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[6] Alternatively, we can think of the matched pair without an outside production opportunity as being involved in continual negotiations by which the expected value of side payments net out to be zero.

payment type j agent receives. Three qualitatively different types of meetings can result from the random matching process: (i) an unmatched employer and an unmatched worker meet and draw a production opportunity, (ii) a matched agent and an unmatched agent meet and draw a production opportunity, and (iii) a matched employer and a matched worker meet and draw a production opportunity. We begin by describing the equilibrium outcome of the bargaining for each of these three types of meetings, taking V_i and V_0 as given. Later, we will analyze how these values are determined in equilibrium.

(i). An unmatched employer meets an unmatched worker.

Suppose an unemployed worker and an employer with a vacancy draw an opportunity for each to produce y_k. Since both are unmatched, the outside option to each agent is V_0. This case is illustrated in Figure 1, where we have named the two agents involved in this meeting A and B.

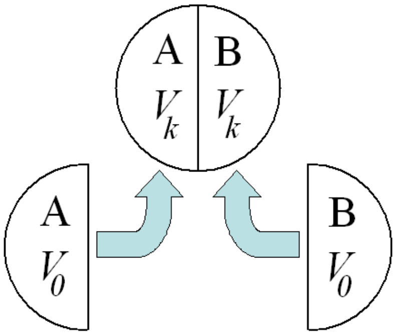

Figure 1: An unmatched employer meets an unmatched worker.

The bargaining unfolds as follows:

Subgame 1. With probability a half, the employer makes a take-it-or-leave-it offer X_{AB}^k to the worker in order to maximize her own utility (which minimizes his partner's utility) subject to the constraint that his partner will accept. Then $X_{AB}^k = V_0$, and the offer is accepted by the partner.

Subgame 2. With the same probability, the worker makes an offer $X_{BA}^{k} = V_0$ to the employer which is again accepted.

Let Π_j be the expected payoff to agent $j = A, B$ and Γ_j be her expected gain. For this case we have $\Pi_A = \Pi_B = \frac{1}{2}V_0 + \frac{1}{2}(2V_k - V_0) = V_k$, and

$$\Gamma_A = \Gamma_B = V_k - V_0. \tag{6}$$

In this symmetric situation the expected value of the side payment is zero, and both unmatched agents enjoy the same capital gains to becoming matched.

(ii). An matched agent meets an unmatched agent.

Suppose agent B, who is currently in a match of productivity y_i with agent A, meets agent C –who is unmatched– and they draw a productive opportunity y_k. This situation is illustrated in Figure 2.

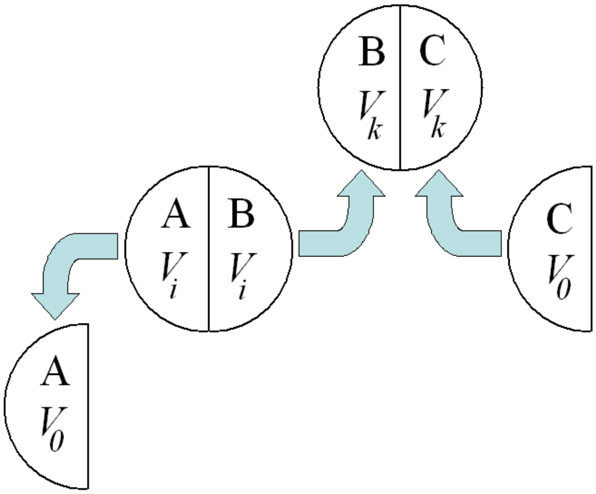

Figure 2: A matched agent meets an unmatched agent.

The bargaining proceeds as follows:

Subgame 1. With probability a half, B makes a take-it-or-leave-it offer to A or C. This offer involves payoffs as well as a proposal to engage in joint production. If B was to offer (continued) joint production to A, he would offer A her minimum acceptable payoff, $X^k_{BA} = V_0$. A would accept the offer and B's payoff from continued production with A would be $2V_i - V_0$. Alternatively, if B offers joint production to C, then he will offer C a payoff equal to her minimum acceptable level, $X^k_{BC} = V_0$. C will accept the offer and B's payoff would be $2V_k - V_0$. So clearly, if $V_k > V_i$ then B offers C to produce together, she accepts, and the payoffs to A, B and C will be V_0, $2V_k - V_0$, and V_0 respectively. Conversely, if $V_i > V_k$, then B offers A to continue to produce together, she accepts, and the payoffs to A, B and C will be V_0, $2V_i - V_0$, and V_0.

Subgame 2. With probability another half, A and C simultaneously make offers to B. Because A's outside option is the value of being unmatched, V_0, the maximum A is willing to offer to B to continue matching with productivity y_i is $2V_i - V_0$, (this offer leaves A with a payoff of V_0). Similarly, the maximum C is willing to offer B in order to form a new match with productivity y_k is $2V_k - V_0$. Since A and C take each other's offer as given, the competition becomes Bertrand, so A offers B's payoff to be $X^i_{AB} = \min(2V_i - V_0, 2V_k - V_0 + \varepsilon)$, and C offers B's payoff to be $X^k_{CB} = \min(2V_i - V_0 + \varepsilon, 2V_k - V_0)$, where ε is an arbitrarily small positive number. Thus, if $V_k > V_i$, then B accepts C's offer to form a new match and the payoffs to A, B and C will be V_0, $2V_i - V_0$ and $2V_k - 2V_i + V_0$ respectively. On the other hand, if $V_i > V_k$, then B accepts A's offer to continue the existing match and the payoffs to A, B and C will be $2V_i - 2V_k + V_0$, $2V_k - V_0$ and V_0.

Notice that regardless of whether it is B who makes the take-it-or-leave-it offer to A or C (subgame 1), or A and C who make the offers to B (subgame 2), B leaves A for C for sure if and only if $V_k > V_i$; that is when the value of the new match exceeds the value of the existing match. The expected capital gains for this case are:

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \end{bmatrix} = \begin{bmatrix} -(V_i - V_0) \\ V_k - V_0 \\ V_k - V_i \end{bmatrix}, \text{ if } V_i < V_k. \tag{7}$$

Notice that through the side payment of transferable utility, the expected gains to the agents who form the new match is equal to the capital gains to their new partner instead of the own capital gains: the gains to B and C are $V_k - V_0$ and $V_k - V_i$ respectively.

On the other hand, if the value of the existing match exceeds the value of the new match, $V_i > V_k$, then regardless of whether it is B or A and C who make the offers, B preserves the match with A, and the expected gains are

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \end{bmatrix} = \begin{bmatrix} -(V_k - V_0) \\ V_k - V_0 \\ 0 \end{bmatrix}, \text{ if } V_k < V_i. \tag{8}$$

Although the current match is not destroyed, the old partner, A, has to transfer the expected value of utility $V_k - V_0$ to B in order to persuade him to stay in the current match. The reason for this transfer is that $V_k - V_0$ is the expected gain for B to form a new match with C (see (7)), so it is also the opportunity cost for B to continue the existing match.

(iii). A matched employer meets a matched worker.

Suppose agent B and agent C meet and draw a productive opportunity y_k. The situation now is that B is currently in a match of productivity y_i with agent A, while C, is currently in a match of productivity y_j with agent D. This case is illustrated in Figure 3.

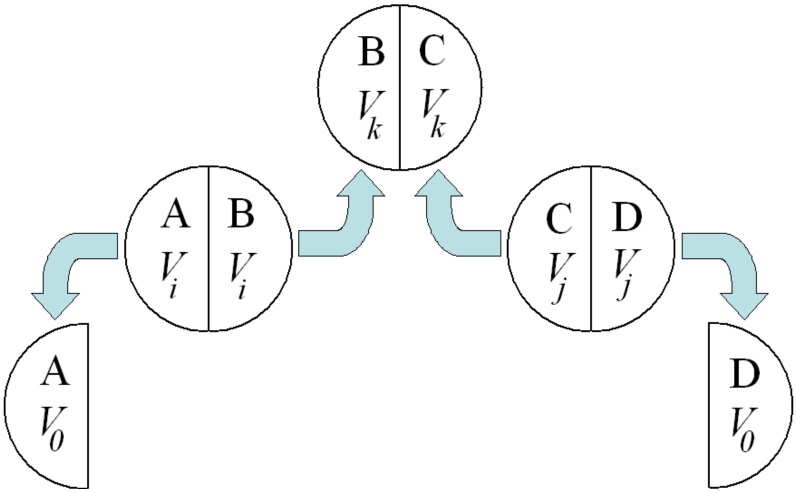

Figure 3: A matched employer meets a matched worker.

The bargaining procedure is as follows:

Subgame 1. With probability a half, A and C simultaneously make offers to B. C also makes a take-it-or-leave-it offer to his existing partner D, and this offer is contingent on his offer to B being rejected. C makes the smallest acceptable offer to D, and since D has no other productive opportunities, his proposed payoff to D is equal to the value of being unmatched, V_0. The resulting payoff to C from continuing to match with D is $2V_j - V_0$, which constitutes the opportunity cost for C to form a new match. Thus the maximum C is willing to offer B is $2V_k - (2V_j - V_0)$. Because A's opportunity cost of continuing to match is the value of being unmatched, V_0, the maximum A is willing to offer B is $2V_i - V_0$. Since this valuation is positive, A will want to make sure that B finds her offer acceptable, and for this she must ensure that B's payoff is at least as large as V_0. Therefore, A offers B's payoff to be $X^i_{AB} = Max\{V_0, Min[2V_i - V_0, 2V_k - (2V_j - V_0) + \varepsilon]\}$ and C offers B's payoff to be $X^k_{CB} = Min[2V_i - V_0 + \varepsilon, 2V_k - (2V_j - V_0)]$ for an arbitrarily small positive ε. Then, B will accept C's offer to form the new match if and only if $2V_k - (2V_j - V_0) > 2V_i - V_0$, or $V_k + V_0 > V_i + V_j$, i.e., the sum of the values of the new match and the unmatched exceeds the sum of the values of the existing matches.

If $V_k < V_i + V_j - V_0$, then A and B preserve their match and whether or not A may have to offer B a side payment depends on whether the new potential match of B and C is better or worse than C's current match. If the new potential match is better (i.e. $V_j < V_k$), then C is willing to offer B as much as $2V_k - (2V_j - V_0) > V_0$ to convince him to leave A, and therefore A has to "bid C away" by giving B a side payment equal to C's valuation of B. However, if $V_k < V_j$, then C is willing to offer B no more than $V_0 + 2(V_k - V_j) < V_0$. But since B can always get V_0 on his own, in this case C's offer poses no threat to A who only has to transfer utility V_0 to B to convince him to preserve their current match.

Subgame 2. With probability another half, B and D simultaneously make offers to C. B also makes an offer to his existing partner, A, and this offer is contingent on his offer to C being rejected. The analysis is identical to that of subgame 1 up to a relabelling so we omit it. (To get the equilibrium payoffs simply replace A with D, B with C, and i with j in the payoffs of subgame 1.)

In the two possible sequences of bargaining (subgame 1 and subgame 2) we see that B and C abandon their old partners to form a new match for sure if and only if the sum of the value of the new match and the unmatched exceeds the sum of two existing matches. Without loss of generality, assume $V_j > V_i$. Then the expected equilibrium gains are:

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} -(V_i - V_0) \\ V_k - V_j \\ V_k - V_i \\ -(V_j - V_0) \end{bmatrix}, \text{ if } V_i + V_j - V_0 < V_k \tag{9}$$

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} -(V_k - V_j) \\ V_k - V_j \\ V_k - V_i \\ -(V_k - V_i) \end{bmatrix}, \text{ if } V_i < V_k < V_i + V_j - V_0 \tag{10}$$

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_k - V_i \\ -(V_k - V_i) \end{bmatrix}, \text{ if } V_i < V_k < V_j \tag{11}$$

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ if } V_k < V_i. \tag{12}$$

In (9), when B and C form a new match, the equilibrium expected side payment is such that the expected gains to each of them is equal to the capital gains to the new partner, instead of their own capital gain.[7] In (10), although the existing matches continue, the old partner must on average pay her current partner his opportunity cost of giving up the option to form a new match. In (11), because the value of the new potential match is not as large as the value of existing match between C and D, A has no need to pay a side payment to B on average in order to persuade him to stay in the existing match. But in expectation, D still needs to pay a side payment to C in order to preserve their valuable match. In (12) the value of the new potential match between B and C is so small that on average A does not have to make a side payment to B and D does not have to make a side payment to C.

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[7] If B and C were to form a new match and there were no side payements, then B would gain $V_k - V_i$ and C would gain $V_k - V_j$, but the equilibrium side payments imply that these gains are swaped: B gains $V_k - V_j$ and C gains $V_k - V_i$. So when a new match is formed, the agent who is currently in the better match enjoys a larger capital gain.

We summarize the main features of the bargaining outcomes in Proposition 1. The proof of parts *(a)* and *(b)* follows from the previous discussion. Part *(c)* is proved in the Appendix which also contains a graphical analysis of the bargaining procedure.

Proposition 1 *For given value functions, the matching decisions and side payments are uniquely determined in the symmetric competitive matching equilibrium through the sequence of bilateral bargaining. Moreover,*
(a) When two agents find an opportunity to form a new match, whether or not they form the new match abandoning their existing matches (if any) depends on whether or not the sum of the values of new match and the unmatched exceeds the total value of the existing matches.
(b) Through the side payment, the expected net gain to the agent who forms a new match is equal to the capital gains of the new partner (instead of his own capital gains).
(c) The equilibrium outcomes (and expected outcomes) induced by the sequence of bilateral bargaining lie in the core.

In the equilibrium, the agents expected payoffs satisfy the following Bellman equations:

$$\begin{aligned} rV_i - \dot{V}_i &= y_i + \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \left[\phi_{ij}^k (V_k + s_{ji}^k - V_i) + (1 - \phi_{ij}^k) \widehat{z}_{ij}^k \right] \\ & \quad - \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \left[\widehat{\phi}_{ij}^k \, (V_i - V_0) + (1 - \widehat{\phi}_{ij}^k) z_{ij}^k \right] - \delta \, (V_i - V_0) \end{aligned}$$

for $i = 1, ..., N$, and

$$rV_0 - \dot{V}_0 = \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \phi_{0j}^k (V_k - V_0 + s_{j0}^k).$$

Here, type i agent's choice of whether or not to form a new match with type j agent is represented by $\phi_{ij}^k \in [0, 1]$. Type i agent's value function also depends upon his existing partner's choices, represented by $\widehat{\phi}_{ij}^k$ and $\widehat{z}_{ij}^k$. We are using s_{ij}^k to denote the net expected side payment that the agent in the type i match who met an agent in a type j match offers her to convince her to form a new match with productivity y_k. Note that $s^k_{ji} = -s^k_{ij}$. Also, we let z^k_{ij} be the expected side payment that type i agent offers his old partner to persuade her to stay in the old match instead of forming a new type k match with an agent who is currently in a type j match.

A competitive matching equilibrium with bargaining is characterized by a set of value functions, side payments and match formation decisions $(V_i, s^k_{ij}, z^k_{ij}, \phi^k_{ij})^N_{i,j=0,k=1}$ together with a population distribution of partnerships $(n_i)^N_{i=0}$ such that: *(i)* Each agent with the opportunity to make an offer chooses how much side payment to offer to her potential partners, and the recipient of the offer chooses whether to accept or reject, in order to maximize her expected discounted utility, taking the strategies of the other agents and the population distribution of partnerships as given; *(ii)* The strategies of the other agents and the population distribution are equilibrium strategies and distribution.

From part *(a)* of Proposition 1 we know that $\phi^k_{ij} = \phi^k_{ji}$, and that $\phi^k_{ij} = 1$ if $V_k + V_0 > V_i + V_j$, $\phi^k_{ij} = 0$ if $V_k + V_0 < V_i + V_j$, and $\phi^k_{ij} \in [0,1]$ if $V_k + V_0 = V_i + V_j$. And from part *(b)* of Proposition 1 we know that if $\phi^k_{ij} = 1$, then $V_k + s^k_{ji} - V_i = V_k - V_j$. Also using the fact that $\widehat{z}^k_{ij} = z^k_{ij}$ and $\widehat{\phi}^k_{ij} = \phi^k_{ij}$ in a symmetric equilibrium, the value functions reduce to

$$rV_i - \dot{V}_i = y_i + \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{0 \leq \phi^k_{ij} \leq 1} \phi^k_{ij} \left(V_k + V_0 - V_i - V_j\right) - \delta \left(V_i - V_0\right)$$

$$rV_0 - \dot{V}_0 = \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{0 \leq \phi^k_{0j} \leq 1} \phi^k_{0j} \left(V_k - V_j\right).$$

Let us define the value of a match to the pair, $\lambda^c_i = 2V_i$ for $i = 0, 1, ..., N$. Then we find the value of the match to the pair satisfies:

$$r\lambda^c_i - \dot{\lambda}^c_i = 2y_i - \delta(\lambda^c_i - \lambda^c_0) + \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{0 \leq \phi^k_{ij} \leq 1} \phi^k_{ij}(\lambda^c_k + \lambda^c_0 - \lambda^c_i - \lambda^c_j) \tag{13}$$

$$r\lambda^c_0 - \dot{\lambda}^c_0 = \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{0 \leq \phi^k_{0j} \leq 1} \phi^k_{0j}(\lambda^c_k - \lambda^c_j). \tag{14}$$

The competitive matching equilibrium can be summarized by a list $(\lambda_i^c, \phi_{ij}^k, n_i)$ for $i, j = 0, ..., N$ and $k = 1, ..., N$ that satisfies (13), (14), and the laws of motion (1) and (2). Notice that the equilibrium value of the match to the pair satisfies very similar conditions to the ones that the shadow price of the match must satisfy for a social optimum. In fact, conditions (13) and (14) would be identical to (4) and (5), were it not for the fact that in the optimality conditions there is a "2" in front of the contact rate α. This difference is due to a search (or match-formation) externality: in the decentralized economy, an individual agent does not take into account the impact that her decisions to form and destroy matches have on the arrival of opportunities of the other agents. Although the arrival rate of any new opportunity is constant here, the arrival rate of a new opportunity with a particular type of agent is proportional to the measure of agents of that type. Also, whether or not a new match is formed depends not only on the quality of the new potential match, but also on the types of the existing matches. Therefore, the relevant meeting rate is quadratic, because the total number of contacts between type i agents and type j agents is equal to $\alpha n_i n_j$.[8]

The relationship between the equilibrium match values and the planner's shadow prices can also be recasted as follows. Define $\mu_i = \lambda_i - \lambda_0$ and $\mu_i^c = \lambda_i^c - \lambda_0^c$. Then from (4), (5), (13) and (14), we have:

$$(r+\delta)\,\mu_i - \dot{\mu}_i = 2y_i + 2\alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{\tau_{ij}^k, \tau_{0j}^k} \left[\tau_{ij}^k \left(\mu_k - \mu_i - \mu_j\right) - \tau_{0j}^k \left(\mu_k - \mu_j\right)\right] \quad (15)$$

$$2\,(r+\delta)\,\mu_i^c - \dot{\mu}_i^c = 4y_i + 2\alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{\phi_{ij}^k, \phi_{0j}^k} \left[\phi_{ij}^k (\mu_k^c - \mu_i^c - \mu_j^c) - \phi_{0j}^k (\mu_k^c - \mu_j^c)\right]. \quad (16)$$

Observe that if we modify the planner's problem replacing r in (15) with $r' = 2r + \delta$, then the first order conditions of this modified planner's problem are identical to the equilibrium conditions for the competitive matching equilibrium, except that the flow outputs y_i all appear multiplied by half for the planner. But a proportional change of all output levels y_i just induces

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[8] Mortensen (1982) shows that "mating models" in which an agent's decisions affect other agents' meeting probabilities typically fail to achieve the socially optimal allocation due to a search externality.

a proportional change in the paths of the μ_i's, which does not change the choices of $\{\tau_{ij}^k, \tau_{0j}^k\}$ nor the resulting distribution $\{n_i\}_{i=1}^N$. We summarize this result as follows:

Proposition 2 *A competitive matching equilibrium exists. Moreover, all competitive matching equilibria satisfy the first order conditions of a modified social planner's problem, in which the subjective discount rate, r, is replaced by the higher rate $r' = 2r + \delta$, where δ is the exogenous destruction rate of any match. The allocation that solves the modified planner's problem can be decentralized as a competitive matching equilibrium.*

4 A Special Case

Consider the model with a fixed population of employers and $N = 2$. For this case the flow conditions (1) and (2) reduce to

$$\begin{aligned}
\dot{n}_2 &= \alpha\pi\left(n_0^2 + 2n_0 n_1 + n_1^2\phi\right) - \delta n_2 \\
\dot{n}_1 &= \alpha\left(1-\pi\right)n_0^2 - 2\alpha\pi n_0 n_1 - 2\alpha\pi n_1^2\phi - \delta n_1 \\
\dot{n}_0 &= \delta\left(n_1 + n_2\right) + \alpha\pi n_1^2\phi - \alpha n_0^2.
\end{aligned}$$

As long as the value function is increasing in the productivity of the current match ($V_0 < V_1 < V_2$), we know that $\phi_{0j}^2 = 1$ for $j = 0, 1$ and that $\phi_{i2}^k = 0$ for $i = 0, 1, 2$ and $k = 1, 2$. To simplify notation, we are letting $\phi = \phi_{11}^2$ and $\pi = \pi_2$ (thus $\pi_1 = 1 - \pi$). Figure 4 illustrates the worker flows.

The following lemma characterizes the steady state distribution of matches taking as given the separation decision ϕ.

Lemma 1 *A unique steady state distribution of workers exists for any given $\phi \in [0, 1]$. The number of unemployed workers, n_0, solves*

$$\left[\alpha n_0^2 - \delta\left(1-n_0\right)\right]\left(\delta + 2\alpha\pi n_0\right)^2 - \phi\alpha\pi\left[2\delta\left(1-n_0\right) - \alpha\left(1+\pi\right)n_0^2\right]^2 = 0.$$

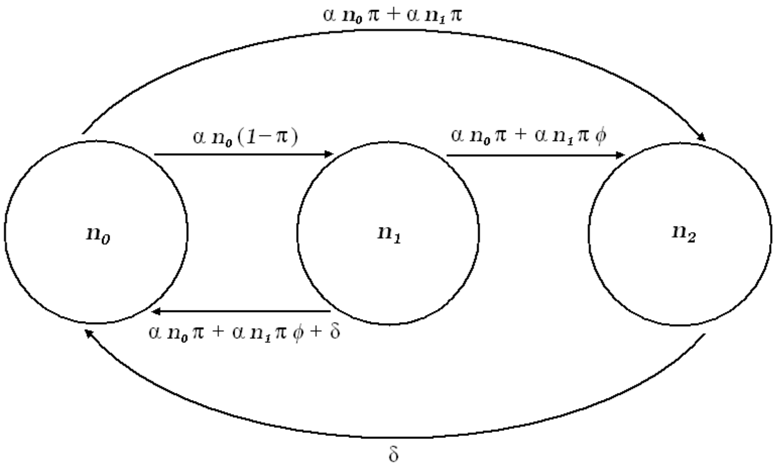

Figure 4: Worker flows for the case of $N = 2$.

The number of workers employed in matches with productivity y_1 *is* $n_1 = \frac{2\delta(1-n_0)-\alpha(1+\pi)n_0^2}{\delta+2\alpha\pi n_0} \equiv f(n_0)$*, and the number of workers employed in matches with productivity* y_2 *is* $n_2 = 1 - n_0 - n_1$*.*

Proof. See the Appendix.

In a stationary equilibrium the value functions satisfy:

$$\begin{aligned} rV_2 &= y_2 - \delta(V_2 - V_0) \\ rV_1 &= y_1 - \delta(V_1 - V_0) + \alpha n_0 \pi (V_2 - V_1) + \alpha n_1 \pi \phi (V_2 + V_0 - 2V_1) \\ rV_0 &= \alpha n_0 [\pi(V_2 - V_0) + (1-\pi)(V_1 - V_0)] + \alpha n_1 \pi (V_2 - V_1). \end{aligned}$$

From Proposition 1 we know that $\phi = 1$ with certainty if and only if $V_2 + V_0 - 2V_1 > 0$. We can use the Bellman equations to write this inequality as

$$\frac{y_2}{y_1} > 2 - \frac{\alpha[\pi n_1 + (1-\pi) n_0]}{r + \delta + \alpha(n_0 + \pi n_1)}, \tag{17}$$

where n_0 and n_1 are the steady state numbers of matches characterized in Lemma 1. Since the right hand side of (17) is bounded, it is clear that $\phi = 1$ with certainty for y_2/y_1 large enough. In these cases, the agents involved will destroy two middle-productivity matches in order to form a single high-productivity match whenever the opportunity arises. Perhaps more surprisingly, notice that there is always some $x > 0$ such that $\phi = 1$ for all $y_2/y_1 > 2 - x$. That is, there may be instances in which two middle-productivity matches are destroyed to form a single high-productivity match even if this entails a reduction in current output. To find a stationary equilibrium, let $n_i(\phi)$ denote the steady state number of matches of productivity y_i as characterized in Lemma 1. Then define the best-response map $\Phi(\phi) = \frac{y_2}{y_1} + \frac{\alpha[\pi n_1(\phi) + (1-\pi) n_0(\phi)]}{r+\delta+\alpha[n_0(\phi)+\pi n_1(\phi)]} - 2$. From this we see that $\phi = 1$ is an equilibrium if $\Phi(1) > 0$, $\phi = 0$ is an equilibrium if $\Phi(0) < 0$ and $\phi^* \in [0,1]$ is an equilibrium if $\Phi(\phi^*) = 0$. The map Φ is continuous on $[0,1]$, so there always exists a stationary equilibrium. However, an equilibrium is not always unique, leading to the possibility of coordination failure. We can show a sufficient condition for the uniqueness of the steady state competitive equilibrium with $N = 2$ is that $\frac{y_2}{y_1} \leq \frac{1+\pi}{1-\pi}$ or that $\frac{y_2}{y_1} \geq 2$ (thus having $\pi \geq \frac{1}{3}$ guarantees uniqueness). In what follows, we continue the discussion for the case of unique equilibrium.

Given (17), Proposition 2 tells us that the social planner chooses to destroy a pair of matches of productivity y_1 to create a single match of productivity y_2 if and only if

$$\frac{y_2}{y_1} > 2 - \frac{2\alpha\left[\pi n_1 + (1-\pi) n_0\right]}{r + \delta + 2\alpha\left(n_0 + \pi n_1\right)}, \tag{18}$$

with n_0 and n_1 given by Lemma 1. Notice that also here, there are instances in which the planner chooses to destroy two matches of productivity y_1 to create a single match of productivity y_2 at the cost of reducing current output. Both in the competitive equilibrium and in the planner's solution the basic logic for this result goes as follows. Although unmatched agents generate zero current output, they generate a positive expected discounted value of output. Hence for some parametrizations (e.g. y_2/y_1 slightly below 2), the planner may choose to reduce current output as a form of investment, in order to increase future output. From a static point of view, this may come as a surprise since unmatched agents are unproductive; but from the planner's dynamic perspective, unmatched agents are a valued input in the matching process that makes production possible. This intuition can be formalized by noticing that both (17) and (18) approach $y_2/y_1 > 2$ as r becomes large. The higher the degree of impatience, the less willing the planner is to trade off current for future production.

From (17) and (18) we also learn that failing to internalize the search externality makes atomistic agents less willing to destroy middle matches relative to the planner. The reason is that the shadow value the planner assigns to a pair of unmatched agents is larger than their value in the competitive equilibrium (because the planner also imputes as part of their return the fact that the unmatched pair helps other agents climb the productivity ladder). Alternatively, recall that from Proposition 2 we know that the competitive matching equilibrium corresponds to a modified planner's economy with higher discount rate $r' = 2r + \delta$. Thus the modified planner is less willing to trade off current for future output. Consequently, the modified planner (or agents in the competitive matching equilibrium) is less willing to trade two matches of productivity y_1 for two agents in a match of productivity y_2 and two unmatched agents. Figure 5 illustrates the difference between the relevant destruction margins in the efficient and the competitive solutions. On the horizontal axis is r, a measure of impatience, and on the vertical axis y_2/y_1, the relevant measure of inequality in instantaneous productivities. Notice that the (n_0, n_1) pair that appears in (17) is identical to that in (18) and is independent of y_1, y_2 and r. (See Lemma 1.) The solid lines with the higher and lower intercepts are conditions (17) and (18) at equality respectively. As in the competitive economy, we know that for the social planner's economy $\tau^2_{0j} = 1$ for $j = 0, 1$; that $\tau^k_{i2} = 0$ for $i = 0, 1, 2$ and $k = 1, 2$ and therefore we use τ to denote τ^2_{11}, the only nontrivial decision.

Double breaches occur in the competitive equilibrium only for parametrizations that lie above the higher solid line. In contrast, the planner implements double breaches for parametrizations that lie above the *lower* solid line. For any given degree of impatience r, the competitive and the efficient allocations coincide only if the flow productivity differential y_2/y_1 is either large enough (i.e. above the higher solid line) or small enough (below the lower solid line). For intermediate values (i.e. those that lie between the two solid lines) the allocations differ: relative to the efficient benchmark, matches of productivity y_1 are too stable in the

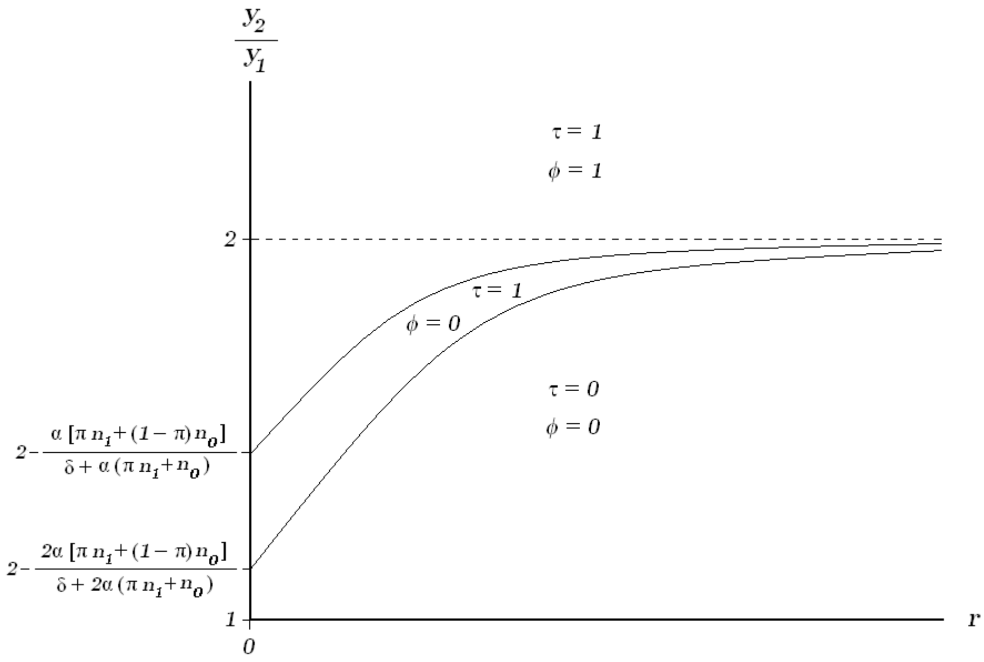

Figure 5: Destruction regions for the case with $N = 2$.

competitive economy.

It is possible to design policies that bring the competitive allocation in line with the planner's. For example, suppose every agent receives a payoff $b > 0$ while unmatched, and that this transfer is paid for by levying a tax T from every match.[9] The balanced-budget condition is $bn_0 = T\,(n_1 + n_2)$. The Bellman equations for the competitive economy become

$$\begin{aligned}
r\hat{V}_2 &= y_2 - T - \delta(\hat{V}_2 - \hat{V}_0) \\
r\hat{V}_1 &= y_1 - T - \delta(\hat{V}_1 - \hat{V}_0) + \alpha n_0 \pi(\hat{V}_2 - \hat{V}_1) + \alpha n_1 \pi \phi(\hat{V}_2 + \hat{V}_0 - 2\hat{V}_1) \\
r\hat{V}_0 &= b + \alpha n_0 \left[\pi(\hat{V}_2 - \hat{V}_0) + (1-\pi)\,(\hat{V}_1 - \hat{V}_0)\right] + \alpha n_1 \pi(\hat{V}_2 - \hat{V}_1).
\end{aligned}$$

Notice that for a given destruction decision ϕ, the stationary distribution of agents across states is still as described in Lemma 1. However, now $\phi = 1$ with certainty if and only if

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[9] For the discussion of this section we will ignore the issue of exactly how a government may be able to collect taxes from agents in a random matching economy, as well as why the same government is unable to facilitate the matching process.

$\hat{V}_2 + \hat{V}_0 - 2\hat{V}_1 > 0$, which can be rewritten as

$$\frac{y_2 - T - b}{y_1 - T - b} > 2 - \frac{\alpha\left[\pi n_1 + (1-\pi)\, n_0\right]}{r + \delta + \alpha\left(n_0 + \pi n_1\right)}.$$

Using the budget constraint the above condition becomes

$$\frac{y_2 - \frac{T}{n_0}}{y_1 - \frac{T}{n_0}} > 2 - \frac{\alpha\left[\pi n_1 + (1-\pi)\, n_0\right]}{r + \delta + \alpha\left(n_0 + \pi n_1\right)}. \tag{19}$$

Observe that if we let $T = T^*$, where

$$T^* = \frac{\alpha n_0 \left(r+\delta\right)\left[\pi n_1 + (1-\pi)\, n_0\right]}{\left[r + \delta + 2\alpha\left(n_0 + \pi n_1\right)\right]\left(r + \delta + \alpha\pi n_0\right)} y_1,$$

then (18) and (19) coincide. In other words, the compensation $b^* = \frac{n_1+n_2}{n_0} T^*$ makes agents internalize the search externality in the competitive matching equilibrium and implements the same destruction decisions as the planner's. Quite intuitively, note that b^* approaches zero as either $r \to \infty$ or $y_1 \to 0$.

The model has clear predictions regarding individual agents' employment histories and the various attributes of different types of jobs. For example, a job of productivity y_2 is not only better paid, but also more stable than a job of productivity y_1. The first observation is immediate because $y_2 > y_1$ (and, in fact, also $V_2 > V_1$). The second follows from the fact that the expected time until a worker gets displaced is $\frac{1}{\delta}$ for a job of productivity y_2 and $\frac{1}{\delta + \alpha\pi(n_0 + \phi n_1)}$ for a job of productivity y_1. Displacement from a job with productivity i is associated with a capital loss equal to $V_i - V_0$, and it takes workers some time to climb back up to a job of productivity equal or higher to the one they were displaced from. For example, suppose a worker is displaced from a job of productivity y_1 (i.e. his match is either hit by the exogenous destruction shock δ, or his employer fires him in order to form a new match of productivity y_2 with another worker). The expected time it takes this worker to find a job at least as good as the one he lost is $\frac{1}{\alpha(n_0 + \phi\pi n_1)}$. Note that the degree of inequality (say as measured by $V_i - V_j$) as well as the shapes of the various hazard rates depend crucially on the separation decisions ϕ. Therefore, we can expect these variables to vary systematically across economies with different labor-market policies that affect this endogenous destruction margin.

We can also construct the theoretical counterparts to the usual empirical measures of job and worker flows. Let JC, JD, WR and WT denote job creation, job destruction, worker reallocation and worker turnover in the stationary equilibrium. Then we have

$$\begin{aligned} JC &= \alpha\left(n_0+\pi n_1\right)n_0 \\ JD &= \alpha\pi\left(n_0+\phi n_1\right)n_1+\delta\left(n_1+n_2\right) \\ WR &= \alpha n_0 n_0+2\alpha n_0 n_1\pi+\alpha n_1 n_0\pi+2\alpha n_1 n_1\pi\phi+\delta\left(n_1+n_2\right) \\ WT &= \alpha n_0 n_0+2\alpha n_0 n_1\pi+2\alpha n_1 n_0\pi+3\alpha n_1 n_1\pi\phi+\delta\left(n_1+n_2\right). \end{aligned}$$

Job creation includes all those unmatched employers who meet and start productive relationships with either unmatched or matched workers. Job destruction consists of all those filled jobs which become unfilled. This occurs every time an employed worker quits to form a better match with another employer and also when the match is destroyed for exogenous reasons. It can be verified that, naturally, $JC - JD = 0$ since the net employment change is zero in the steady state. Worker reallocation counts the number of workers who change state. In the first term are the number of unemployed workers who fill vacant jobs. In the second term are the unemployed workers who contact a filled job and get hired. The "2" multiplying this term accounts for the change of state of the previously employed worker who gets displaced. The third term represents the number of previously employed workers who contact a vacant job and quit to form a more productive relationship. The fourth term accounts for the number of workers who are employed and quit to form a new match with an employer who was previously matched to another worker, as well as for the corresponding displaced workers. The number of workers who change state (i.e. become unemployed) for exogenous reasons are accounted for in the last term. The measure of worker turnover counts the total number of accessions and separations over all employers.

Notice that the gross job and worker flows satisfy:

$$\begin{aligned} WR &= JC + JD + \alpha\pi n_0 n_1 + \alpha\pi n_1 n_1 \phi \\ WT &= WR + \alpha\pi n_0 n_1 + \alpha\pi n_1 n_1 \phi. \end{aligned}$$

In the model –as in the data– gross worker reallocation is larger than gross job reallocation, $JC + JD$. Instances of "replacement hiring" are behind this discrepancy, since job creation and destruction are unchanged when a firm fires a worker to replace him with an unemployed one. But also, in economies in which $\phi > 0$, there is yet another reason for worker reallocation in excess of job reallocation, since when a matched employer and an employed worker decide to form a new match the worker reallocation count increases by 2 while job reallocation only increases by 1 (job creation is unchanged by this transition).[10] Workers who experience job-to-job transitions get counted twice in the aggregate measure of worker turnover, so the number of job-to-job transitions, $\alpha\pi n_0 n_1 + \alpha\pi n_1 n_1 \phi$, is the amount by which worker turnover exceeds worker reallocation.

5 Employment Protection

In this section we introduce two broad sets of employment protection policies. The first consists of policies specifying that the agent who breaks up a match is to compensate her old partner for the loss she inflicts on him. The second set of policies differ in that the party that initiates the separation must pay the "government" a firing tax, and the government then offers the displaced agent a compensation.

5.1 Firing Compensation

We begin by studying the bargaining procedure in the presence of a policy that specifies the agent who leaves a relationship must pay compensatory damages to the old partner. Because

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[10] Several recent empirical studies argue that distinguising between job and worker flows is essential for a complete characterization of aggregate labor-market dynamics. See Fallick and Fleischman (2001), Nagypál (2003) and Stewart (2002).

firing compensation are a pure transfer among partners, it does not change the total surplus of the alternative matches of all the members involved. One expects that the Coase Theorem will hold, so that the decision to form a new match continues to be privately efficient; i.e. efficient for all the parties involved in the meeting, given the value functions. More subtle is the effect that firing compensation will have on the value functions themselves. Consider the single-breach situation illustrated in Figure 2 and let $T^k_{i0} \leq V_i - V_0$ be the compensation that B must pay A should he leave to form a new productive relationship with C. As usual, the bargaining procedure is composed of two subgames.

Subgame 1. With probability a half, B makes a take-it-or-leave-it offer specifying continuation payoffs as well as a proposal to engage in joint production to either A or C. If B was to offer continued joint production to A, he would offer A her minimum acceptable payoff, $X^k_{BA} = V_0 + T^k_{i0}$. Agent A would accept the offer and B's payoff from continued production with A would then be $2V_i - V_0 - T^k_{i0}$. Alternatively, if B was to offer joint production to C he would offer her $X^k_{BC} = V_0$, her minimum acceptable continuation value. C would accept this offer and B's payoff after paying the firing compensation to A would be $2V_k - V_0 - T^k_{i0}$. If $V_k > V_i$ then B will choose to leave A and form a new match with C. The payoffs to A, B and C will be $V_0 + T^k_{i0}$, $2V_k - V_0 - T^k_{i0}$, and V_0 respectively. Alternatively, if $V_k < V_i$, then B will offer continued production to A and the payoffs to A, B and C will be $V_0 + T^k_{i0}$, $2V_i - V_0 - T^k_{i0}$, and V_0.

Subgame 2. With probability another half, A and C simultaneously make offers to B. Since A's outside option is now $V_0 + T^k_{i0}$, she is willing to offer B no more than $2V_i - V_0 - T^k_{i0}$. On the other hand, the maximum C is willing to offer B is $2V_k - V_0$. Therefore A offers B a continuation payoff $X^i_{AB} = \max\left[\min\left(2V_i - V_0 - T^k_{i0}, 2V_k - V_0 - T^k_{i0} + \varepsilon\right), V_0\right]$ and C's offer is for B's continuation payoff to be $X^k_{CB} = \min\left(2V_k - V_0 - T^k_{i0}, 2V_i - V_0 - T^k_{i0} + \varepsilon\right)$ where ε is an arbitrarily small positive number.[11] If $V_k > V_i$ then B forms a new match with C and the

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[11] The compensation T^k_{i0} appears subtracting from the second argument of the "min" in X^k_{AB} and from the first argument of the "min" in X^k_{CB} because when C transfers $2V_k - V_0$ to B, if B matches with C he only gets $2V_k - V_0 - T^k_{ij}$ after settling the firing compensation with A. The "max" in X^k_{CB} ensures that A never offers B

payoffs to A, B and C are $V_0 + T_{i0}^k$, $2V_i - V_0 - T_{i0}^k$, and $2V_k - (2V_i - V_0)$ respectively. Conversely, if $V_k < V_i$ then B stays matched to A and the payoffs to A, B and C are $2V_i - \left(2V_k - V_0 - T_{i0}^k\right)$, $2V_k - V_0 - T_{i0}^k$, and V_0 respectively.

In both subgames B leaves A for sure if and only if $V_k > V_i$. In this case the expected capital gains are

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \end{bmatrix} = \begin{bmatrix} -\left(V_i - V_0 - T_{i0}^k\right) \\ V_k - V_0 - T_{i0}^k \\ V_k - V_i \end{bmatrix}.$$

If $V_0 + T_{i0}^k \leq V_k \leq V_i$, then A and B preserve their match. The expected capital gains by

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \end{bmatrix} = \begin{bmatrix} -\left(V_k - V_0 - T_{i0}^k\right) \\ V_k - V_0 - T_{i0}^k \\ 0 \end{bmatrix}.$$

If $V_k < V_0 + T_{i0}^k$, then B remains matched to A and is unable to extract a positive expected side payment from her: all agents' continuation payoffs remain unchanged and nobody experiences capital gains or losses. Note that if the policy requires the partner who leaves to pay *fully compensatory* damages to her old partner, i.e. if $T_{i0}^k = V_i - V_0$ for all i and k, then $\Gamma_B = \Gamma_C = V_k - V_i$ and $\Gamma_A = 0$ in those cases in which B chooses to form a new match with C. In those cases in which $V_0 + T_{i0} \leq V_k \leq V_i$, A transfers $\Gamma_B = V_k - V_i$ to B and persuades him to preserve their current match.

Next, we consider the double-breach situation illustrated in Figure 3 and let $T_{ij}^k \leq V_i - V_0$ be the compensation that B must pay A should he leave to form a new productive relationship with C. Similarly, $T_{ji}^k \leq V_j - V_0$ is the compensation that C must pay D should she leave to form a new productive relationship with B.

Subgame 1. With probability a half, A and C simultaneously make offers to B. C also makes a take-it-or-leave-it offer to his existing partner D, and this offer is contingent on his offer to B being rejected. C makes the smallest acceptable offer to D, namely $V_0 + T_{ji}^k$. The resulting payoff to C from continuing to match with D is $2V_j - V_0 - T_{ji}^k$, which constitutes the opportunity cost for C to form a new match. Thus the maximum payoff C is willing to

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a continuation payoff below V_0 even in instances where C's valuation of B, i.e. $2V_k - V_0$, is less than $V_0 + T_{i0}^k$.

assign to B is $2V_k - T^k_{ij} - T^k_{ji} - (2V_j - V_0 - T^k_{ji})$. If B "fires" A, then A's continuation payoff is $V_0 + T^k_{ij}$. Thus the maximum A is willing to offer B is $2V_i - V_0 - T^k_{ij}$. Since this valuation is positive (recall that $T^k_{ij} \leq V_i - V_0$), A will want to make sure that B finds her offer acceptable, and for this she must ensure that B's payoff is at least as large as V_0. Therefore, A offers B a continuation payoff $X^i_{AB} = Max\{V_0, Min[2V_i - V_0 - T^k_{ij}, 2V_k - (2V_j - V_0) - T^k_{ij} + \varepsilon]\}$ and C offers B's payoff to be $X^k_{CB} = Min[2V_k - (2V_j - V_0) - T^k_{ij}, 2V_i - V_0 - T^k_{ij} + \varepsilon]$ for an arbitrarily small positive ε. Then, B will accept C's offer to form the new match if and only if $V_k + V_0 > V_i + V_j$.

Subgame 2. With probability another half, B and D simultaneously make offers to C. B also makes an offer to his current partner A, and this offer is contingent on his offer to C being rejected. This subgame is identical to subgame 1 up to a relabeling so we omit the analysis.

In the two possible sequences of bargaining (subgame 1 and subgame 2) B and C abandon their old partners to form a new match for sure if and only if the sum of the value of the new match and the unmatched exceeds the sum of two existing matches. The equilibrium expected gains are:

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} -(V_i - V_0 - T^k_{ij}) \\ V_k - V_j - T^k_{ij} \\ V_k - V_i - T^k_{ji} \\ -(V_j - V_0 - T^k_{ji}) \end{bmatrix}, \text{ if } V_i + V_j - V_0 < V_k$$

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} -\max(V_k - V_j - T^k_{ij}, 0) \\ \max(V_k - V_j - T^k_{ij}, 0) \\ \max(V_k - V_i - T^k_{ji}, 0) \\ -\max(V_k - V_i + T^k_{ji}, 0) \end{bmatrix}, \text{ if } V_k \leq V_i + V_j - V_0.$$

If $V_k < V_j + T^k_{ij}$, then B remains matched to A and is unable to use his meeting with C to extract a side payment from A. Similarly, C is unable to extract a side payment from D if $V_k < V_i + T^k_{ji}$. Note that if the policy requires the partner who breaks the match to pay *fully compensatory* damages to her old partner, i.e. if $T^k_{ij} = V_i - V_0$ for all i, j and k, then A and D never suffer any capital losses (or equivalently, B and C never experience capital gains). By construction, the policy ensures A and D suffer no losses when their matches are destroyed by their partners, but as it turns out, this policy will also spare them from having to make side payments to prevent their respective partners from leaving in those cases in which B and C have the option of forming a match of type k with $V_k \leq V_i + V_j - V_0$.

To conclude, we return to the value functions to see how the policies affect the equilibrium payoffs associated with each state:

$$\begin{aligned} rV_i - \dot{V}_i &= y_i + \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \left[\phi_{ij}^k (V_k - V_j - T_{ij}^k) + (1 - \phi_{ij}^k) \widehat{z}_{ij}^k \right] \\ & \quad - \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \left[\widehat{\phi}_{ij}^k (V_i - V_0 - T_{ij}^k) + (1 - \widehat{\phi}_{ij}^k) z_{ij}^k \right] - \delta \left(V_i - V_0 \right) \end{aligned}$$

for $i = 1, ..., N$, and

$$rV_0 - \dot{V}_0 = \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \phi_{0j}^k \left(V_k - V_j \right).$$

In a symmetric equilibrium $\widehat{\phi}_{ij}^k = \phi_{ij}^k$, and these expressions reduce to (13) and (14). We summarize this result as follows.

Proposition 3 *Policies that require the partner who breaks the relationship to (either partially or fully) compensate the old partner are completely neutral: they have no effect on payoffs nor on match formation and dissolution decisions.*

Thus firing compensation not only has no effect on the new match formation decisions given the value functions, but also has no effect on the value functions themselves.

5.2 Firing Taxes

We now report the main results for the case of a policy specifying that the agent who leaves a relationship must pay a tax. (See the Appendix for details.) We still use $T_{ij}^k \leq V_i - V_0$ to denote the tax that an agent currently in a type i match who forms a new type k match with an agent who was previously in a type j match must pay for separating from his current partner. The difference with the previous case is that this “firing tax” is paid out to some external party (e.g. a “government”); i.e. it is not directly transferred to the old partner. We allow for the possibility that the breached against partner receives compensation $S_{ij}^k \le V_i - V_0$ from the government. The key is that although T_{ij}^k and S_{ij}^k will typically be related through some overall government budget constraint, they need not be equal to each other.[12]

Firing taxes will in general alter the match formation and destruction decisions. Summarizing, in the single-breach situation of Figure 2, B will destroy his match with A to form a new one with C if and only if

$$2V_k + V_0 - (T_{i0}^k - S_{i0}^k) > 2V_i + V_0.$$

And in the double-breach situation of Figure 3 B and C leave their current partners if and only if

$$2V_k + 2V_0 - (T_{ij}^k + T_{ji}^k - S_{ij}^k - S_{ji}^k) > 2V_i + 2V_j.$$

Imposing high firing taxes on the agents who "fire" their partners tends to make existing matches more stable while generous government transfers to the displaced agents makes existing matches more likely to be destroyed. What matters for the creation and destruction decisions is how much all the members involved in a meeting (including the agents who get fired) pay in net to the government. So if the government increases the payments of the private agents, say by imposing a more stringent administrative procedure for firing, then the simultaneous creation (of new matches) and destruction of (old) matches will decrease further. To conclude, turn to the value functions to see how the policies affect the equilibrium payoffs associated with each state. Using the equilibrium break up rules and focusing on a symmetric equilibrium $\widehat{\phi}_{ij}^k = \phi_{ij}^k$, the Bellman equations are:

$$\begin{aligned} rV_i - \dot{V}_i &= y_i - \delta\left(V_i - V_0\right) \\ &\quad + \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{0 \le \phi_{ij}^k \le 1} \phi_{ij}^k \left[V_k + V_0 - V_i - V_j - \frac{1}{2}(T_{ij}^k + T_{ji}^k - S_{ij}^k - S_{ji}^k) \right] \\ rV_0 - \dot{V}_0 &= \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{0 \le \phi_{0j}^k \le 1} \phi_{0j}^k \left[V_k - V_j - \frac{1}{2}(T_{j0}^k - S_{j0}^k) \right]. \end{aligned}$$

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[12] For example we may have $S_{ij}^k > T_{ij}^k$ if the government collects other taxes in addition to the firing taxes, or $S_{ij}^k < T_{ij}^k$ if the proceeds from the firing taxes are also used to pay for other programs.

The policy that requires each agent who breaks a match to directly compensate her old partner corresponds to the special case with $T_{ij}^k = S_{ij}^k$ for all i, j and k and is completely neutral as shown previously.[13]

6 Free Entry

So far we have been assuming constant (and equal) populations of employers and workers. In this section we generalize the formulation by allowing for free entry of employers. Let m_j be the number of employers in state j; we still use n_i to denote number of workers in state i. Since there is one-to-one matching we have $m_i = n_i$ for all $i \geq 1$ but n_0 (the number of unemployed workers) may be larger or smaller than m_0 (the number of vacant employers). We assume that a worker contacts an employer in state j at rate αm_j, while an employer contacts a random worker in state i at rate αn_i.[14]

The measure of workers in each state evolves according to:

$$\begin{aligned} \dot{n}_i &= \alpha \pi_i \sum_{j=0}^{N} \sum_{k=0}^{N} n_j m_k \tau_{jk}^i - \alpha n_i \sum_{j=0}^{N} \sum_{k=1}^{N} m_j \pi_k \tau_{ij}^k \\ &\quad - \alpha m_i \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \tau_{ji}^k - \delta n_i \qquad (20) \\ \dot{n}_0 &= \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} n_i m_j \pi_k \tau_{ij}^k + \delta \sum_{j=1}^{N} n_j - \alpha n_0 m_0 \sum_{k=1}^{N} \pi_k \tau_{00}^k. \qquad (21) \end{aligned}$$

The first term on the right hand side of (20) is the flow of new matches of productivity y_i created by all types of workers and employers. The second term is the total flow of matches

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[13] The idea that government-mandated transfers between the employer and the worker can be offset by private contracts between the parties goes back to Lazear (1990). Lazear also notes that severance pay effects are neutral only when the payment made by the employer is received by the worker, and not if third-party intermediaries receive or make any of the payments.

[14] This formulation implies that the total number of meetings is given by a quadratic matching technology $\xi(N_e, N_w) = \alpha N_e N_w$, where N_e is the total numbers of employers and N_w the total number of workers. In our formulation, $N_w = 1$ and $N_e = 1 - n_0 + m_0$. We have also considered and will be reporting results for the case in which the aggregate meeting technology is instead given by a function $\xi(N_e, N_w)$ which is monotonic in both arguments and homogeneous of degree one. In this alternative formulation an employer contacts a random worker at rate $\alpha(N_e) = \xi(1, 1/N_e)$ and worker contacts a random employer at rate $N_e \alpha(N_e)$. But note that even if we adopt a matching technology that is linearly homogeneous in the aggregate populations, the matching process will still be effectively quadratic in the relevant stocks of workers and employers, $(n_i)_{i=0}^N$ and $(m_j)_{j=0}^N$.

with productivity y_i destroyed endogenously when the worker "quits" to form a new match with another employer. The third term represents those matches with productivity y_i that are destroyed when the employer "fires" the worker in order to form a new match with another worker. The last term is the flow of matches dissolved exogenously. On the right hand side of (21), the first term is the flow of workers who are displaced when their employers decide to break up their current match to form a new match with another worker. The workers whose matches are destroyed exogenously are accounted for by the second term. The last term is the flow of new matches created by unemployed workers and unmatched employers.

Before turning to the competitive matching equilibrium we pose the planner's problem. The planner chooses $\tau_{ij}^k \in [0,1]$ and $m_0 \geq 0$ to maximize the discounted value of aggregate output

$$\int e^{-rt} \left[\sum_{i=1}^{N} 2y_i n_i - C\left(m_0\right) \right] dt$$

subject to the flow constraints (20) and (21) and initial conditions for n_0 and n_i and m_i for $i = 1, ..., N$. Note that while unmatched employers incur a cost $C\left(m_0\right)$, with $C' > 0$ and $C'' \geq 0$.[15] Letting λ_i be the shadow price associated with the flow equation of the i^{th} state, the Hamiltonian corresponding to the planner's problem is

$$\begin{aligned} H &= \sum_{i=1}^{N} 2y_i n_i - C\left(m_0\right) - \delta \sum_{i=1}^{N} n_i \left(\lambda_i - \lambda_0\right) \\ &\quad + \alpha \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{N} n_i m_j \pi_k \tau_{ij}^k \left(\lambda_k + \lambda_0 - \lambda_i - \lambda_j\right). \end{aligned}$$

The optimality conditions are:

$$\tau_{ij}^k \begin{cases} = 1 & \text{if } \lambda_k + \lambda_0 > \lambda_i + \lambda_j \\ \in [0,1] & \text{if } \lambda_k + \lambda_0 = \lambda_i + \lambda_j \\ = 0 & \text{if } \lambda_k + \lambda_0 < \lambda_i + \lambda_j \end{cases} \tag{22}$$

and

$$C'\left(m_0\right) \geq \alpha \sum_{i=0}^{N} \sum_{k=1}^{N} n_i \pi_k \tau_{i0}^k \left(\lambda_k - \lambda_i\right) \tag{23}$$

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[15] In Pissarides (2000), $C\left(m_0\right)$ is the "cost of posting vacancies m_0".

with "=" if $m_0 > 0$. Condition (22) is familiar from the previous analysis. The left hand side of condition (23) is the marginal cost of an unmatched employer (or the marginal cost of "opening a vacancy"), and the right hand side is the expected return from having an additional vacancy (note that $\lambda_k - \lambda_i$ is the capital gain to the planner from creating a new match of quality y_k by matching a vacancy to a worker previously in a match of quality y_i, while $\alpha n_i \pi_k \tau_{i0}^k$ is the probability that this capital gain is realized). Focusing on a solution with a positive measure of unmatched employers, (23) can be rewritten as[16]

$$C'(m_0) = \alpha \sum_{i=0}^{N} \sum_{k=1}^{N} n_i \pi_k \tau_{i0}^k (\lambda_k - \lambda_i). \tag{24}$$

For $i \geq 1$ the Euler equations are:

$$\begin{aligned} r\lambda_i - \dot{\lambda}_i &= 2y_i - \delta(\lambda_i - \lambda_0) + \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k (\tau_{ij}^k + \tau_{ji}^k)(\lambda_k + \lambda_0 - \lambda_i - \lambda_j) \\ &\quad + \alpha m_0 \sum_{k=1}^{N} \pi_k \tau_{i0}^k (\lambda_k - \lambda_i) + \alpha n_0 \sum_{k=1}^{N} \pi_k \tau_{0i}^k (\lambda_k - \lambda_i). \end{aligned}$$

The right hand side of this condition is readily interpreted as the flow return to the planner from allocating an additional worker to a match of quality y_i. A (worker in a) match of type i yields output $2y_i$ and a capital loss $\lambda_i - \lambda_0$ in the event of an exogenous break-up. The remaining terms represent the expected capital gains from matching that are generated by an additional match of type i. An additional match of quality i generates expected capital gains both directly, when it climbs up the productivity ladder itself, and indirectly, by increasing the probability that an agent in a match of quality j will meet an agent in a match of quality i and climb up the

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[16] For the constant-returns matching case (20), (21), H and (22) are as given in the text, while condition (24) becomes

$$\begin{aligned} C'(m_0) &= \alpha \sum_{i=0}^{N} \sum_{k=1}^{N} n_i \pi_k \tau_{i0}^k (\lambda_k - \lambda_i) + \alpha'(\theta) m_0 \sum_{i=0}^{N} \sum_{k=1}^{N} n_i \pi_k \tau_{i0}^k (\lambda_k - \lambda_i) \\ &\quad + \alpha'(\theta) \sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} n_i n_j \pi_k \tau_{ij}^k (\lambda_k + \lambda_0 - \lambda_i - \lambda_j). \end{aligned}$$

In all these expression α should be interpreted as $\alpha(N_e)$, an employer's contact rate. The last term represents "congestion externalities".

productivity ladder himself. Naturally, the planner internalizes both these returns. (As we saw in the model with fixed populations, only the direct return enters the agent's calculations in the decentralized economy.) The fourth term represents the expected capital gain that accrues to the planner when the worker in a match of type i meets an unmatched employer. Similarly, the fifth term is the expected capital gain to the planner from having the employer in a match of type i meet an unemployed worker. Similarly, for $i = 0$ we have:

$$r\lambda_0 - \dot{\lambda}_0 = \alpha m_0 \sum_{k=1}^{N} \pi_k \tau_{00}^k \left(\lambda_k - \lambda_0\right) + \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k \tau_{0j}^k \left(\lambda_k - \lambda_j\right).$$

The right hand side can again be interpreted as the marginal return of an unemployed worker. The first term is the expected capital gain the unemployed worker generates in the event she matches with an unmatched employer. The second term is the expected capital gain in the event she matches with a matched employer.[17] Using (24), which holds as long as $m_0 > 0$, and collecting terms we arrive at:[18]

$$\begin{aligned} r\lambda_0 - \dot{\lambda}_0 &= -C'\left(m_0\right) + \alpha\left(m_0 + n_0\right) \sum_{k=1}^{N} \pi_k \tau_{00}^k \left(\lambda_k - \lambda_0\right) \\ &\quad + \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k (\tau_{0j}^k + \tau_{j0}^k) \left(\lambda_k - \lambda_j\right). \end{aligned}$$

From (22) we see that $\tau_{ij}^k = \tau_{ji}^k$ except possibly for the case of randomized strategies. Using this symmetry ($m_i = n_i$ for $i \geq 1$), we can write the Euler equations together with the optimality

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[17] For the constant-returns matching case the Euler equation associated with n_i is as in the text, while the one associated with n_0 is

$$\begin{aligned} r\lambda_0 - \dot{\lambda}_0 &= \alpha m_0 \sum_{k=1}^{N} \pi_k \tau_{00}^k \left(\lambda_k - \lambda_0\right) + \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k \tau_{0j}^k \left(\lambda_k - \lambda_j\right) \\ &\quad -\alpha'\left(\theta\right) m_0 \sum_{i=0}^{N} \sum_{k=1}^{N} n_i \pi_k \tau_{i0}^k \left(\lambda_k - \lambda_i\right) \\ &\quad -\alpha'\left(\theta\right) \sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} n_i n_j \pi_k \tau_{ij}^k \left(\lambda_k + \lambda_0 - \lambda_i - \lambda_j\right). \end{aligned}$$

[18] This expression remains unchanged in the formulation with constant returns to scale.

conditions (22) more compactly as

$$\begin{aligned} r\lambda_i - \dot{\lambda}_i &= 2y_i - \delta\left(\lambda_i - \lambda_0\right) + \alpha\left(m_0 + n_0\right)\sum_{k=1}^{N}\pi_k \max_{0\leq\tau_{i0}^k\leq 1} \tau_{i0}^k\left(\lambda_k - \lambda_i\right) \\ &\quad + 2\alpha\sum_{j=1}^{N}\sum_{k=1}^{N} n_j\pi_k \max_{0\leq\tau_{ij}^k\leq 1} \tau_{ij}^k\left(\lambda_k + \lambda_0 - \lambda_i - \lambda_j\right) \qquad (25) \\ r\lambda_0 - \dot{\lambda}_0 &= -C'\left(m_0\right) + \alpha\left(m_0 + n_0\right)\sum_{k=1}^{N}\pi_k \max_{0\leq\tau_{00}^k\leq 1} \tau_{00}^k\left(\lambda_k - \lambda_0\right) \\ &\quad + 2\alpha\sum_{j=1}^{N}\sum_{k=1}^{N} n_j\pi_k \max_{0\leq\tau_{0j}^k\leq 1} \tau_{0j}^k\left(\lambda_k - \lambda_j\right). \qquad (26) \end{aligned}$$

Conditions (25) and (26) are very similar to the first order conditions for the model with a fixed number of employers. In particular, note that (25) and (26) reduce to (4) and (5) respectively if we set $C' = 0$ and $m_0 = n_0$. But more generally, in this formulation we have an additional unknown, m_0, and (24) provides the additional optimality condition.

Next, we characterize the competitive matching equilibrium using the bargaining procedure we introduced in Section 3. Figures 1, 2 and 3 still describe the basic types of meetings.

(i). An unmatched employer meets an unemployed worker.

We begin with the situation illustrated in Figure 1, that is a bargaining situation in which neither the employer nor the worker have outside opportunities. Agent A is an unemployed worker, B an unmatched employer and M_k represents the value of a match of type k in the competitive matching equilibrium. We use V_0 and J_0 to denote the values of an unemployed worker and an unmatched employer respectively.As usual, the bargaining sequence is composed of:

Subgame 1. With probability one half, the worker makes a take-it-or-leave-it offer $X_{AB}^k = J_0$ which is accepted by the employer.

Subgame 2. With probability one half, the employer makes a take-it-or-leave-it offer $X_{BA}^k = V_0$ which is accepted by the worker.

The expected payoffs to the worker A and the employer B are $\Pi_A = V_0 + \frac{1}{2}\left(M_k - M_0\right)$ and

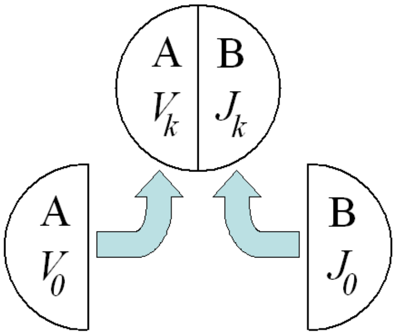

Figure 6: An unmatched employer meets an unemployed worker.

$\Pi_B = J_0 + \frac{1}{2}(M_k - M_0)$ respectively, where $M_0 = V_0 + J_0$. So when A and B first meet and form a match, their expected capital gains are

$$\Gamma_A = \Gamma_B = \frac{1}{2}(M_k - M_0).$$

We think of a matched pair with no outside production opportunities as being involved in continuous negotiations of the type illustrated by Figure 1. Output is continuously divided among the partners in such as way so that the worker's continuation payoff is

$$V_k = V_0 + \frac{1}{2}(M_k - M_0)$$

and the employer's is

$$J_k = J_0 + \frac{1}{2}(M_k - M_0).$$

(ii). An matched employer meets an unemployed worker.

Employer B, who is currently hiring worker A in a match with productivity y_i, meets an unemployed worker C and they draw a production opportunity y_k. This situation is illustrated in Figure 7.

Subgame 1. With probability a half, B makes a take-it-or-leave-it offer specifying continuation payoffs as well as a proposal to engage in joint production to either A or C. If B was to offer (continued) joint production to A, he would offer A her minimum acceptable payoff, $X_{BA}^k = V_0$.

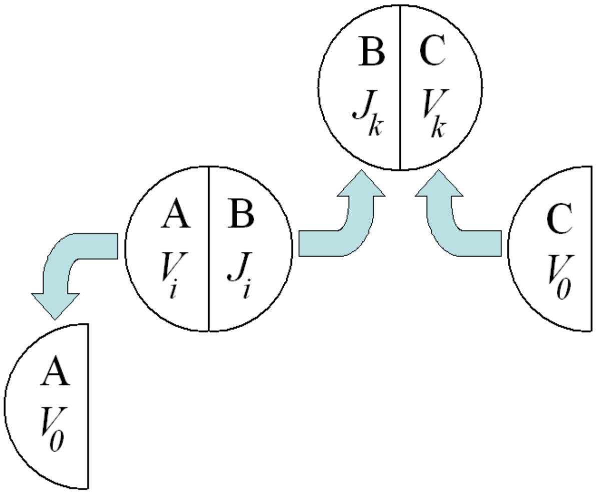

Figure 7: A matched employer meets an unemployed worker.

Worker A would accept the offer and B's payoff from continuing the match with A would be $M_i - V_0$. Alternatively, if B offers joint production to worker C, then he would also offer C her minimum acceptable continuation payoff $X^k_{BC} = V_0$; C would accept and B's continuation payoff from forming a new match with C would be $M_k - V_0$. Thus B will fire A to form a new type k match with C if and only if $M_k > M_i$. In this case the payoffs to A, B and C are V_0, $M_k - V_0$ and V_0 respectively. Conversely, if $M_k < M_i$, then B offers continued production to A, she accepts and the payoffs to A, B and C are V_0, $M_i - V_0$ and V_0.

Subgame 2. With probability another half, A and C simultaneously make offers to B. Worker A offers B's payoff to be $X^i_{AB} = \min(M_i - V_0, M_k - V_0 + \varepsilon)$ and worker C offers B's payoff to be $X^k_{CB} = \min(M_k - V_0, M_i - V_0 + \varepsilon)$, where ε is an arbitrarily small positive number. If $M_k > M_i$ then B accepts C's offer to form a new match, and the payoffs to A, B and C are V_0, $M_i - V_0$, and $M_k - M_i + V_0$. Conversely, if $M_k < M_i$ then B accepts A's offer to continue their match and the payoffs to A, B and C are $M_i - M_k + V_0$, $M_k - V_0$ and V_0 respectively.

In both subgames B fires A to form a new match with C if and only if $M_k > M_i$. The expected capital gains are

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -(M_i - M_0) \\ M_k - M_0 \\ M_k - M_i \end{bmatrix} \text{ if } M_k > M_i \tag{27}$$

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -(M_k - M_0) \\ M_k - M_0 \\ 0 \end{bmatrix} \text{ if } M_k < M_i. \tag{28}$$

(iii). An unmatched employer meets an employed worker.

Worker B who is employed with A meets C, an unmatched employer. The analysis of this case amounts to a relabelling of the previous one so we just note that the expected capital gains Γ_A, Γ_B and Γ_C are given by (27) and (28).

(iv). A matched employer meets an employed worker.

Suppose that worker B and employer C meet and have the option to form a new match of type k. The circumstances are now that B is currently in a match of type i with employer A, and C is in a match of type j with worker D. This situation is illustrated in Figure 8.

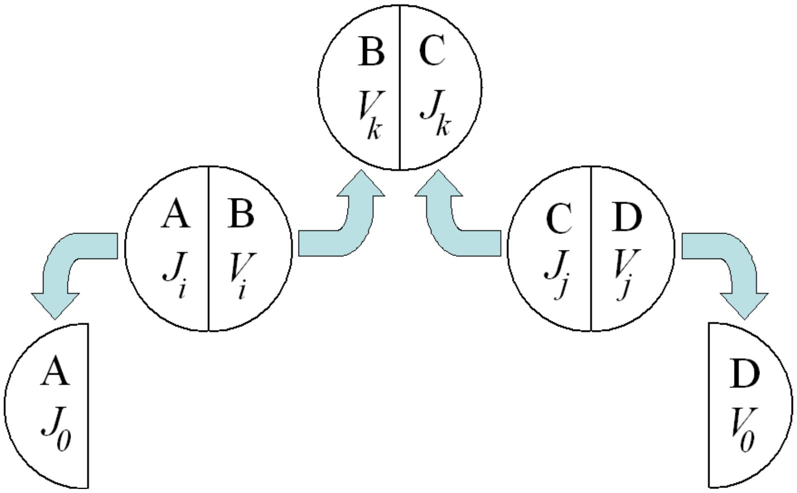

Figure 8: An employed worker meets a matched employer.

Subgame 1. With probability a half, the two employers A and C simultaneously make offers to B. Employer C also makes a take-it-or-leave-it offer to her current worker D, and this offer is contingent on her offer to worker B being rejected. By the usual arguments, A offers B's continuation payoff to be $X^i_{AB} = \max\{\min[M_i - J_0, M_k - (M_j - V_0) + \varepsilon], V_0\}$. Similarly, C offers B's continuation payoff to be $X^k_{BC} = \min[M_k - (M_j - V_0), M_i - J_0 + \varepsilon]$.

Subgame 2. With probability a half, the two workers B and D simultaneously make offers to employer C. Worker B also makes a take-it-or-leave-it offer to her employer A. The analysis follows closely that of subgame 1.

In both subgames B and C leave their current partners to form a new match of type k if and only if $M_i + M_j - M_0 < M_k$. Suppose, without loss of generality, that $M_i < M_j$. If $M_j < M_k < M_i + M_j - M_0$, then B and C stay in their current matches and extract strictly positive expected side payments from their respective partners. If $M_i < M_k < M_j$, then the existing matches are preserved but only C is able to extract a strictly positive expected side payment from her partner. This meeting does not generate enough bargaining power for B to be able to extract resources from A. Finally, if $M_k < M_i$, then B and C stay in their current matches and neither of them is able to benefit from the meeting. The equilibrium expected gains are:

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -(M_i - M_0) \\ M_k - M_j \\ M_k - M_i \\ -(M_j - M_0) \end{bmatrix}, \text{ if } M_i + M_j - M_0 < M_k$$

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -(M_k - M_j) \\ M_k - M_j \\ M_k - M_i \\ -(M_k - M_i) \end{bmatrix}, \text{ if } M_i < M_k < M_i + M_j - M_0$$

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ M_k - M_i \\ -(M_k - M_i) \end{bmatrix}, \text{ if } M_i < M_k < M_j$$

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ if } M_k < M_i.$$

Given the equilibrium outcomes of the bargaining procedure, in the equilibrium the expected payoffs to an unemployed worker and an unmatched firm satisfy the following Bellman equations:

$$
\begin{aligned}
rV_0 &= \alpha m_0 \sum_{k=1}^{N} \pi_k \phi_{00}^{k} \frac{M_k - M_0}{2} + \alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \phi_{0j}^{k} \frac{M_k - M_j}{2} && (29) \\
rJ_0 &= -c + \alpha n_0 \sum_{k=1}^{N} \pi_k \phi_{00}^{k} \frac{M_k - M_0}{2} + \alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \phi_{j0}^{k} \frac{M_k - M_j}{2}. && (30)
\end{aligned}
$$

Here each employer who posts a vacancy pays $c = C'(m_0)$, while filled employers do not have to pay anything (say because production itself is free advertisement to attract workers).[19] As usual, ϕ_{ij}^{k} denotes the probability with which a match of type i and a match of type j are destroyed to form a new match of type k in the equilibrium outcome of the bargaining procedure. For $i = 1, ..., N$ and letting w_i denote the worker's wage while employed in a match of type i, the value of a worker in a match of type i is

$$
\begin{aligned}
rV_i = {} & w_i - \delta (V_i - V_0) \\
& + \alpha m_0 \sum_{k=1}^{N} \pi_k \left[\phi_{i0}^{k} \frac{M_k - M_0}{2} + (1 - \phi_{i0}^{k}) \frac{M_k - M_0}{2} \right] \\
& - \alpha n_0 \sum_{k=1}^{N} \pi_k \left[\phi_{0i}^{k} \frac{M_i - M_0}{2} + (1 - \phi_{0i}^{k}) \frac{M_k - M_0}{2} \right] \\
& + \alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \left[\phi_{ij}^{k} \frac{M_k - M_j}{2} + (1 - \phi_{ij}^{k}) \max\left(\frac{M_k - M_j}{2}, 0 \right) \right] \\
& - \alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \left[\phi_{ji}^{k} \frac{M_i - M_0}{2} + (1 - \phi_{ji}^{k}) \max\left(\frac{M_k - M_j}{2}, 0 \right) \right]. \qquad (31)
\end{aligned}
$$

---

[19] If $C(m_0)$ is strictly convex, profit $cm_0 - C(m_0)$ is distributed to the owners of the scarce factor in the vacancy-posting technology. This profit will not affect the labor market because the utility function is linear.

Similarly, the value of an employer in a match of productivity y_i is:

$$\begin{aligned} rJ_i &= 2y_i - w_i - \delta\left(J_i - J_0\right) \\ &\quad +\alpha n_0 \sum_{k=1}^{N} \pi_k \left[\phi_{0i}^{k} \frac{M_k - M_0}{2} + (1-\phi_{0i}^{k}) \frac{M_k - M_0}{2}\right] \\ &\quad -\alpha m_0 \sum_{k=1}^{N} \pi_k \left[\phi_{i0}^{k} \frac{M_i - M_0}{2} + (1-\phi_{i0}^{k}) \frac{M_k - M_0}{2}\right] \\ &\quad +\alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \left[\phi_{ji}^{k} \frac{M_k - M_j}{2} + (1-\phi_{ji}^{k}) \max\left(\frac{M_k - M_j}{2}, 0\right)\right] \\ &\quad -\alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \left[\phi_{ij}^{k} \frac{M_i - M_0}{2} + (1-\phi_{ij}^{k}) \max\left(\frac{M_k - M_j}{2}, 0\right)\right]. \qquad (32) \end{aligned}$$

Since $\phi_{ij}^{k} = \phi_{ji}^{k}$, adding (32) to (31) and (30) to (29) respectively imply

$$\begin{aligned} rM_i &= 2y_i - \delta\left(M_i - M_0\right) + \frac{\alpha\left(m_0 + n_0\right)}{2} \sum_{k=1}^{N} \pi_k \phi_{i0}^{k} \left(M_k - M_i\right) \\ &\quad \alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \phi_{ij}^{k} \left(M_k + M_0 - M_i - M_j\right). \qquad (33) \\ rM_0 &= -c + \frac{\alpha\left(m_0 + n_0\right)}{2} \sum_{k=1}^{N} \pi_k \phi_{00}^{k} \left(M_k - M_0\right) \\ &\quad +\alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \phi_{0j}^{k} \left(M_k - M_j\right). \qquad (34) \end{aligned}$$

Since there is free entry of employers, any equilibrium with a positive measure of unmatched employers must be such that the expected return to an unmatched employer is just enough to cover the entry cost:

$$C'\left(m_0\right) = c = \frac{\alpha}{2} \sum_{i=0}^{N}\sum_{k=1}^{N} n_i \pi_k \phi_{i0}^{k} \left(M_k - M_i\right). \qquad (35)$$

If we compare (33), (34) and (35) with (25), (26) and (24) we see that –just as in the case with equal and fixed populations of employers and workers– the planner's first-order conditions and the equilibrium conditions differ only in that in his calculations the planner imputes an "effective" contact rate equal to 2α while α is the contact rate to an individual agent. Alternatively, if we replace the subjective interest rate of the social planner r, with $r' = 2r + \delta$, then again, the first order conditions corresponding to the modified planner's problem correspond to one of the competitive matching equilibria. If the equilibrium is unique, then the equilibrium allocation is identical to that of the modified social planner's economy.

7 Discussion

In this section we discuss how our paper relates to the existing theoretical literature on labor market matching models with on-the-job search. Burdett (1978) adds on-the-job search to the single-agent search decision problem faced by a worker who samples wages from an exogenous distribution. Mortensen (1978) studies the relationship between the nature of the wage bargaining problem between a worker and an employer and their choices of on-the-job search intensities. He observes that the search intensities the employer and worker choose in a Nash equilibrium of the noncooperative game are too high relative to those that would be chosen jointly to maximize the value of the match. He then explores the ability of two alternative mechanisms to improve efficiency when agents choose their search strategies noncooperatively. The mechanisms do not require direct monitoring, but rely on both agents' ability to commit to future actions. The first is an *ex ante* agreement by each partner to make a counteroffer when the other receives an attractive alternative matching opportunity. The second is an *ex ante* agreement to fully compensate the other partner as a precondition for separation. Relative to the joint wealth maximizing strategy, both partners search too much in the noncooperative Nash equilibrium under the mechanism with commitment to counteroffer. But under the commitment to fully compensate the partner in case separation, the Nash noncooperative equilibrium delivers the pair of search strategies that maximize the joint surplus.

Diamond and Maskin (1979) extends Mortensen (1978) by embedding the search problem of the single partnership in an equilibrium model with many potential partnerships. They study the steady-state equilibria of a model in which agents are randomly paired in a costly search process to carry out a single productive project. As in our setup, agents are *ex ante* homogeneous, but matches are heterogeneous *ex post* and utility is transferable. A difference is that once matched, their agents decide whether or not to continue searching and only after partners have stopped searching is the project completed and both agents exit the market. The interesting situations arise when a matched agent finds the option to break the current match to form a new one. In their language, a *single breach (of contract)* occurs when a matched agent forms a new match with an unmatched agent, while a *double breach* takes place when two matched agents leave their partners to form a new match. The two key differences from our work are that in that model *(i)* agents always split the match surplus symmetrically, and *(ii)* in anticipation of possible breaches, contracts may provide for compensation or "damages" to be paid to the breached-against partner, which requires that agents have the ability to commit to future actions or else that "courts" exogenously enforce these contracts. Diamond and Maskin show that if the partner who breaks the match is required to fully compensate the breached-against partner for the loss she suffers, then as in our competitive matching equilibrium, the two individuals with the option to form a new match find it in their interest to breach precisely when by doing so they increase the sum of the expected payoffs of the four parties involved in the meeting. The difference is that our competitive matching equilibrium achieves this outcome through a more flexible bargaining process involving side payments, without requiring that agents be able to commit to compensate their partners in case of future breaches.

In Diamond and Maskin (1979) agents match to produce one time. In some unpublished notes, Diamond and Maskin (1981) extend that framework to allow for continuous production. Their physical environment corresponds to the special case of our economy with $N = 2$. In this version they continue to assume that partners split the matching surplus symmetrically and that when a partner separates she must pay the breached-against partner compensatory damages, and explore some properties of a steady-state equilibrium in which single breaches occur but double breaches do not.[20]

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[20] If we set $N = 2$, and conjecture that $\tau_{ij}^{k} = \tau_{ji}^{k}$ and $\tau_{ij}^{i} = 0$ for $j \leq i$, set $\tau_{00}^{1} = \tau_{00}^{2} = \tau_{10}^{2} = 1$, and specialize the analysis to an equilibrium with $\tau_{11}^{2} = 0$, then (1)-(2) would reduce to the flow equations on page 4 of Diamond and Maskin (1981).

The model in Burdett, Imai and Wright (2004) also has *ex ante* homogeneous agents, *ex post* heterogeneous matches, costly search, and agents who while matched decide whether to search or not. They consider two setups. In the first setup, they assume that once two agents make contact, they cannot observe the realization of their prospective match productivity unless they drop their current partners (if they have any).[21] Utility may be interpreted to be transferable or not in this setup. For this version of the model they provide a full characterization of the equilibrium set and its welfare properties. The second setup allows agents to keep the option to stay with their current partners after observing the realization of the match quality with a prospective partner. They lay out the model with two types of matches and argue that their main results (e.g., multiplicity and efficiency properties of equilibria) are robust to this generalization. This second setup relies on the assumption that utility is nontransferable.[22] This must be so because if utility were transferable, then matched agents would attempt to counter their partners' outside offers just as they do in our model. So, although the physical environment of Burdett, Imai and Wright (2004) is essentially the same as ours, their analysis is quite different because they make assumptions that rule out the multilateral breach situations that are an essential part of our notion of equilibrium.

Burdett and Mortensen (1998) develop an influential on-the-job search model with *ex ante* homogeneous populations of employers and workers.[23] Employers are assumed to post and

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[21] This assumption makes their model extremely tractable by eliminating "composition effects": The gains from forming a match of a given quality are the same regardless of the state of the other partner, so the value functions are independent of the endogenous distribution of match qualities among actual relationships. The fact that payoffs depend on the distribution of characteristics of potential partners is a feature that arises naturally in our model and in many other matching models, both with *ex post* match heterogeneity and on-the-job search (e.g., Diamond and Maskin (1979, 1981)) and with *ex ante* heterogeneity, even with no on-the-job search (e.g., Burdett and Coles (1997) and Shimer and Smith (2000, 2001)).

[22] The on-the-job search model of Cornelius (2003) also assumes utility is nontransferable, but differs from Burdett, Imai and Wright (2004) in that agents are *ex ante* heterogeneous, search is costless both on and off the job, and the meeting technology is quadratic.

[23] The Burdett-Mortensen model was originally developed to explain wage dispersion among homogeneous workers and relate it to employer size, but has by now been extended in many ways and applied to study a wide range of issues, both empirically and theoretically. Van den Berg and Ridder (1998) and Bontemps, Robin and Van den Berg (1999, 2000) are examples of papers that have structurally estimated the model. Theoretical extensions and applications include Burdett and Coles (2003) and Burdett, Lagos and Wright (2004). See Manning (2003) and Mortensen (2003) for other applications and more references.

commit to wages, have access to a constant returns to scale production technology, and may employ any number of workers at the posted wage. Whenever an employed worker meets an employer with a posted wage higher than her current wage, she quits to join the new employer's workforce. Therefore, employers who post low wages experience high quit rates and have smaller workforces in the steady state. By requiring that steady-state profit be equated across firms, Burdett and Mortensen derive a nondegenerate equilibrium wage distribution. Note that there is an extreme notion of commitment at work in this model: once the employer has chosen a wage to offer its employees, the assumption is that it cannot be changed. It cannot be raised to counter a worker's outside offer, and it cannot be cut down once the outside offer is gone.

Postel-Vinay and Robin (2000) work out an extension of Burdett and Mortensen (1998) with *ex ante* heterogeneous employers and workers. Employers still have the power to make take-it-or-leave-it offers to workers but are in addition allowed to counter the offers that their workers receive from competing employers. When a worker of productivity ε who is matched to a firm with productivity p contacts a potential employer with productivity p', the employers enter a Bertrand competition for the worker that is ultimately won by the most productive firm. If $p > p'$, then the worker stays with the current employer, who from then on is assumed to be committed to paying her no less than the wage that won the Bertrand competition. If $p < p'$, then the worker quits to go to the higher productivity employer, who is also assumed to pay no less than the winning wage for the duration of the match.[24] Relative to Burdett-Mortensen, the extension of Postel-Vinay and Robin (2000) assumes a weaker form of commitment: Firms still commit not to reduce wages in the future, but can counter outside offers. In a different way, the extension of Coles (2001) also assumes a weaker form of commitment, this time by assuming

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[24] Instead of giving the firm the power to make a worker take-it-or-leave-it offers, in Dey and Flinn (2000) employers and workers in continuing relationships split the match surplus according to the Nash cooperative solution. If an employed worker is contacted by another employer, then the current and prospective employers enter a Bertrand competition for the worker. Again, the employer with higher productivity can always offer the worker higher continuation utility and hence "wins" the worker. From then on, once the worker's outside offer is gone, the assumption is that the match continues to split the surplus according to the Nash solution where the threat point is taken to be the maximal continuation value offered to the worker by the firm that lost the last Bertrand competition to hire him. (Workers who are hired from the unemployment pool bargain with their value of search as threat point until they get a better outside offer while searching on the job.)

firms cannot respond to outside offers but can change wages during times when their workforce has no outside offers outstanding. From this perspective, our paper takes the analysis a step further by modeling agents who cannot commit to any future actions.

Another relevant difference is that in the Burdett-Mortensen approach each employer operates a constant returns to scale production technology that can in principle employ the whole population of workers. So if there are heterogeneous employers, it would be desirable and technologically feasible to have all workers matched to the highest-productivity employer. In contrast, we study the consequences of the opposite assumption to constant returns by assuming that each employer can hire at most one worker. This extreme version of decreasing returns enriches the sets of transitions that employers and workers can engage in, with no loss of tractability. For example, the model delivers endogenous "firing" in addition to endogenous "quits". Also, the limited-capacity assumption allows the model to exhibit instances of replacement hiring as well as situations in which – in the language of the empirical labor flows literature – job reallocation induces worker reallocation and vice versa.

In Pissarides (1994) or Pissarides (2000) employed workers can search on the job, but employers do not (so all quits involve workers taking jobs that were previously vacant), and the wage is assumed to be determined according to a linear surplus splitting rule at all times. Relative to what we do here, a key difference is that both in Pissarides (1994, 2000) and in Shimer (2004) matched employers are not allowed to offer side payments to counter their worker's outside offers, and similarly, a vacant employer who contacts an employed worker cannot make side payments to persuade the worker to quit. Competition involving side payments among all the parties involved in a typical on-the-job search meeting is an essential feature of the equilibrium in the model we develop here. Also, we propose a competitive bargaining procedure to split the gains from trade instead of relying on surplus splitting rules or the Nash axiomatic approach.[25]

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[25] Shimer (2004) points out that in the context of the on-the-job search model of Pissarides (1994, 2000), a simple linear surplus splitting rule is in general not equivalent to the Nash bargaining solution and that adopting the former may lead to pair-wise inefficient outcomes. In addition, Shimer (2004) argues that when the linear splitting rule is replaced by the Nash bargaining solution the model is capable of generating equilibria with wage dispersion even in the case of *ex ante* identical employers and workers.

8 Concluding Remarks

We have developed a model of on-the-job search that has many of the stylized properties of actual labor markets. Worker flows exceed job flows, displaced agents suffer persistent reductions in permanent incomes, job-to-job transitions are common and firms often engage in simultaneous hiring and firing. We have proposed and analyzed a notion of competitive equilibrium based on a particular bargaining procedure and explored its efficiency properties.

Several extensions seem worth pursuing. First, motivated by the observations in Bertola and Rogerson (1997) and Blanchard and Portugal (2001), the model could be used to analyze the effects that employment protection policies have on the amount of worker reallocation in excess of job reallocation. Bertola and Rogerson find that despite higher employment protection in Europe than in the United States, European job turnover rates are not that different from those in the United States, yet there is evidence that worker turnover (and in particular the rate at which workers enter and leave unemployment) is lower in Europe. Blanchard and Portugal report that relative to those in the United States, worker flows are much smaller in Portugal, even for given job flows. In particular, the flow of workers out of employment in Portugal barely exceeds job destruction, and they attribute this to the Portuguese employment protection policies. Our model suggests a simple explanation for these observations: Employment protection policies censor precisely the transitions that cause worker turnover in excess of job turnover, namely, separations resulting from double breaches and from employer-initiated single breaches.

A related issue is that, given the empirical relevance of job-to-job flows, an appropriate assessment of the welfare effects of employment protection policies calls for a model with on-the-job search, perhaps along the lines we have proposed here. Calculating the welfare effects of employment protection policies with a model that does not allow for job-to-job transitions is likely to underestimate the welfare losses from the policy. For example, Blanchard and Portugal assume that all separations (either employer- or worker-initiated) result in the worker being unemployed and the firm vacant. But suppose – as is the case in our model – that separations do not necessarily result in both partners being unmatched. Then policies that discourage separations will tend to have higher overall costs than in an environment where quits necessarily entail an unemployment spell.

At a deeper level, we would also like to understand why employment protection policies exist. In our framework with one-employer-to-one-worker matching and transferable utility, workers and employers are essentially symmetric (even if allowing for free entry of employers introduces a slight asymmetry), and employment protection policies result in no efficiency gains. To explore the rationale behind the existence of employment protection policies, perhaps, we have to introduce some asymmetry, such as that each worker works for one employer while each employer hires several workers. This extension would also be useful to address many empirical issues such as the size distribution of firms or the relationship between firm size and job and worker flows.

A Appendix

Proof of Lemma 1. Let $f\left(n_0\right) \equiv \frac{2\delta(1-n_0)-\alpha(1+\pi)n_0^2}{\delta+2\alpha\pi n_0}$. Combining the $\dot{n}_2 = 0$ and $\dot{n}_0 = 0$ conditions we see that $n_1 = f\left(n_0\right)$. It can be shown that $f' < 0$ on $[0,1]$, so to each $n_0 \in [0,1]$ corresponds a unique n_1. In addition, $f\left(n_0\right) \geq 0$ if $n_0 \leq \overline{\eta}_0$ and $f\left(n_0\right) \leq 1$ if $n_0 \geq \underline{\eta_0}$, where

$$\overline{\eta}_0 = \frac{\sqrt{\delta^2+2\alpha\delta(1+\pi)}-\delta}{\alpha(1+\pi)} \qquad \text{and} \qquad \underline{\eta_0} = \frac{\sqrt{(\delta+\alpha\pi)^2+\alpha\delta(1+\pi)}-(\delta+\alpha\pi)}{\alpha(1+\pi)},$$

with $0 < \underline{\eta_0} < \overline{\eta}_0 < 1$. Let

$$G\left(n_0;\phi\right) \equiv \left[\alpha n_0^2 - \delta\left(1-n_0\right)\right]\left(\delta+2\alpha\pi n_0\right)^2 - \phi\alpha\pi\left[2\delta\left(1-n_0\right)-\alpha\left(1+\pi\right)n_0^2\right]^2.$$

Substituting $n_1 = f\left(n_0\right)$ back into the $\dot{n}_0 = 0$ delivers a single equation in n_0 which can be written as $G\left(n_0;\phi\right) = 0$. Direct calculations reveal that $G(\overline{\eta}_0;\phi) = \alpha\overline{\eta}_{00}^{2} - \delta(1-\overline{\eta}_0) > 0$ for all $\phi \in [0,1]$. Also, $G(\underline{\eta_0};\phi) = \alpha\underline{\eta_0}^2 - \delta(1-\underline{\eta_0}) - \alpha\phi\pi$. Note that an increase in ϕ causes G to shift down uniformly. Therefore, to ensure that $G(\underline{\eta_0};\phi) < 0$ for all ϕ it suffices to guarantee that $G(\underline{\eta_0};0) < 0$. This condition can be written as $\alpha\underline{\eta_0}^2 - \delta(1-\underline{\eta_0}) < 0$, a parametric restriction that is always satisfied. Finally, note that $\left.\frac{\partial G(n_0;\phi)}{\partial n_0}\right|_{G(n_0;\phi)=0} > 0$, which together with the fact that $f' < 0$ implies that the steady state is unique. ■

Bargaining outcomes and the core.

Before proving part *(c)* of Proposition 1 we introduce some notation. Let I denote the set of agents who are directly or indirectly (i.e. through a partner) involved in a meeting. For example, $I = \{A, B, C, D\}$ in the situation illustrated in Figure 3. Within the context of a meeting, an *allocation* is a collection of partnerships. For example, there are two possible allocations for the meeting in Figure 3: $\langle (A,B), (C,D) \rangle$ and $\langle (B,C), (A,D) \rangle$. The first represents the case in which A remains matched to B while C remains matched to D. The second corresponds to the case in which B and C form a new match while A and D become unmatched (or become matched to each other but in state 0).[26] Let $\mathcal{A}_j$ denote the set of all possible allocations in a meeting that

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[26] We ignore other feasible allocations such as $\langle (A,C), (B,D) \rangle$, which would correspond to "break up both matches without forming a new one" because they will play no role in the analysis that follows.

concerns j agents. Then, $\mathcal{A}_2 = \{\langle (A,B) \rangle, \langle (A), (B) \rangle\}$, $\mathcal{A}_3 = \{\langle (A,B), (C) \rangle, \langle (A), (B,C) \rangle\}$ and $\mathcal{A}_4 = \{\langle (A,B), (C,D) \rangle, \langle (B,C), (A,D) \rangle\}$. An allocation $a \in \mathcal{A}_j$ together with a payoff profile $\Pi \in \mathbb{R}^j$ constitute an *outcome* $[a, \Pi]$. For example, $[\langle (A), (B) \rangle, (\Pi_A, \Pi_B)]$ with $\Pi_A = \Pi_B = V_0$ is the outcome corresponding to a situation in which two unmatched agents meet and no match is formed. For any given meeting, a nonempty subset $S \subseteq I$ is called a *coalition.* Let v denote a function that assigns a real number to each coalition S. The number $v(S)$ is called the *worth* of coalition S. Since utility is fully transferable, $v(S)$ summarizes the utility possibility set of coalition S. Intuitively, $v(S)$ is the total utility available to the coalition, which can then be distributed among the coalition members in any way. An outcome $[a, \Pi]$ is *blocked* by a coalition S if there exists a payoff profile $\tilde{\Pi}$ with $\sum_{i \in S} \tilde{\Pi}_i \leq v(S)$ such that $\tilde{\Pi}_i > \Pi_i$ for all $i \in S$. With transferable utility, an outcome $[a, \Pi]$ is blocked by S iff $\sum_{i \in S} \Pi_i < v(S)$. An outcome $[a, \Pi]$ that is feasible for the grand coalition (i.e. such that $\sum_{i \in I} \Pi_i \leq v(I)$) is in the *core* if there is no coalition S that blocks this outcome. With transferable utility, an outcome $[a, \Pi]$ is in the core iff $\sum_{i \in S} \Pi_i \geq v(S)$ for all $S \subseteq I$ and $\sum_{i \in I} \Pi_i \leq v(I)$.

Proof of part *(c)* of Proposition 1. The proof proceeds in three steps.

(Step 1). First consider the case illustrated in Figure 1, where an unemployed worker A and an unmatched employer B meet and have the opportunity to form a match of productivity $y_k > 0$. For this case we have $I = \{A, B\}$, and the list of all possible coalitions is $\{A, B\}$, $\{A\}$, $\{B\}$. The worth of the grand coalition is $v(I) = \max(2V_0, 2V_k) = 2V_k$, while $v(\{A\}) = v(\{B\}) = V_0$. A vector of of payoffs (Π_A, Π_B) lies in the core if and only if *(i)* $\Pi_A + \Pi_B = 2V_k$; and *(ii)* $\Pi_j \geq V_0$ for $j = A, B$. Figure 9 shows the core: it is the segment on the $\Pi_A + \Pi_B = 2V_k$ line that lies between the equilibrium payoffs of subgames 1 and 2 of the bilateral bargaining procedure. Both equilibrium payoffs as well as the expected payoff lie in the core.

(Step 2). Next consider the case illustrated in Figure 2: agent B who is currently in a match of productivity y_i with agent A, meets unmatched agent C and they draw a productive opportunity y_k. Here $I = \{A, B, C\}$ and the list of all possible coalitions is $\{A, B, C\}$, $\{A, B\}$,

$\{A, C\}$, $\{B, C\}$, $\{A\}$, $\{B\}$, $\{C\}$. The corresponding values are $v(I) = \max(2V_i + V_0, 2V_k + V_0)$, $v(\{A, B\}) = 2V_i$, $v(\{A, C\}) = 2V_0$, $v(\{B, C\}) = 2V_k$, $v(\{A\}) = v(\{B\}) = v(\{C\}) = V_0$. Hence a payoff profile $\Pi = (\Pi_A, \Pi_B, \Pi_C)$ belongs to the core if and only if: *(i)* $\Pi_A + \Pi_B + \Pi_C = \max(2V_i + V_0, 2V_k + V_0)$; *(ii)* $\Pi_A + \Pi_B \geq 2V_i$; *(iii)* $\Pi_B + \Pi_C \geq 2V_k$; and *(iv)* $\Pi_j \geq V_0$ for $j = A, B, C$. If $V_k > V_i$ the four conditions can be rewritten as: *(1)* $\Pi_A = V_0$; *(2)* $\Pi_B \geq 2V_i - V_0$; *(3)* $\Pi_B + \Pi_C = 2V_k$; and *(4)* $\Pi_C \geq V_0$. The first panel of Figure 10 illustrates the core for this case; it consists of all the payoffs (V_0, Π_B, Π_C) such that (Π_B, Π_C) lie on the segment of the $\Pi_B + \Pi_C = 2V_k$ line between the equilibrium payoffs of subgames 1 and 2 of the bilateral bargaining procedure. From the figure it is clear that the equilibrium payoffs of both subgames and the expected payoff all belong to the core. Conversely, if $V_k < V_i$, then the four conditions reduce to: *(1')* $\Pi_A \geq V_0$; *(2')* $\Pi_B \geq 2V_k - V_0$; *(3')* $\Pi_A + \Pi_B = 2V_i$; and *(4')* $\Pi_C = V_0$. The second panel of Figure 10 illustrates the core for this case; it consists of all the payoffs (Π_A, Π_B, V_0) such that (Π_A, Π_B) lie on the segment of the $\Pi_A + \Pi_B = 2V_i$ line between the equilibrium payoffs of subgames 1 and 2 of the bilateral bargaining procedure. From the figure it is again clear that the equilibrium payoffs of both subgames and the expected payoff all belong to the core.

(Step 3). Finally, consider the case illustrated in Figure 3: while A and B are in a match of productivity y_i and C and D are in a match of productivity y_j, agents B and C meet and draw a productive opportunity y_k. Here $I = \{A, B, C, D\}$ and the list of all possible coalitions is: $\{A, B, C, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{B, C, D\}$, $\{A, C, D\}$, $\{A, B\}$, $\{C, D\}$, $\{A, C\}$, $\{B, D\}$, $\{B, C\}$, $\{A, D\}$, $\{A\}$, $\{B\}$, $\{C\}$, $\{D\}$. The corresponding values are $v(I) = \max(2V_k + 2V_0, 2V_i + 2V_j)$, $v(\{A, B, C\}) = \max(2V_i + V_0, 2V_k + V_0)$, $v(\{A, B, D\}) = 2V_i + V_0$, $v(\{B, C, D\}) = \max(2V_j + V_0, 2V_k + V_0)$, $v(\{A, C, D\}) = 2V_j + V_0$, $v(\{A, B\}) = 2V_i$, $v(\{C, D\}) = 2V_j$, $v(\{A, C\}) = v\{\{B, D\}\} = v\{\{A, D\}\} = 2V_0$, $v(\{A\}) = v(\{B\}) = v(\{C\}) = v(\{D\}) = V_0$. A payoff profile $\Pi = (\Pi_A, \Pi_B, \Pi_C, \Pi_D)$ is in the core if and only if it satisfies the following inequalities: $\Pi_A + \Pi_B + \Pi_C + \Pi_D = \max(2V_k + 2V_0, 2V_i + 2V_j)$, $\Pi_A + \Pi_B + \Pi_C \geq \max(2V_i + V_0, 2V_k + V_0)$, $\Pi_B + \Pi_C + \Pi_D \geq \max(2V_j + V_0, 2V_k + V_0)$, $\Pi_A +$

$\Pi_B+\Pi_D \geq 2V_i+V_0$, $\Pi_A+\Pi_C+\Pi_D \geq 2V_j+V_0$, $\Pi_A+\Pi_B \geq 2V_i$, $\Pi_C+\Pi_D \geq 2V_j$, $\Pi_B+\Pi_C \geq 2V_k$, $\Pi_A+\Pi_C \geq 2V_0$, $\Pi_B+\Pi_D \geq 2V_0$, $\Pi_A+\Pi_D \geq 2V_0$, $\Pi_j \geq V_0$ for $j = A, B, C, D$. It is straightforward to verify that the equilibrium and expected payoffs of the bilateral bargaining procedure satisfy these fifteen inequalities. ■

We now provide a graphical analysis of the bargaining outcome in a meeting involving four agents. Assume, with no loss of generality, that $V_j > V_i$. We begin analyzing the case in which $V_j + V_i - V_0 < V_k$. For this case it can be shown that any payoff profile in the core must have $\Pi_A = \Pi_D = V_0$, $\Pi_B \geq 2V_i - V_0$, $\Pi_C \geq 2V_j - V_0$, and $\Pi_B + \Pi_C = 2V_k$. A simple two-dimensional figure can still be used to fully characterize the core. This is done in Figure 11.

Next consider the case $V_j < V_k < V_j + V_i - V_0$. It is possible to show that any payoff profile $\Pi = (\Pi_A, \Pi_B, \Pi_C, \Pi_D)$ in the core must satisfy: $V_0 \leq \Pi_A \leq 2(V_i + V_j - V_k) - V_0$, $V_0 + 2(V_k - V_j) \leq \Pi_B \leq 2V_i - V_0$, $V_0 + 2(V_k - V_i) \leq \Pi_C \leq 2V_j - V_0$, $V_0 \leq \Pi_D \leq 2(V_i + V_j - V_k) - V_0$. Since illustrating the core payoffs now requires a three-dimensional diagram, we instead provide a simpler two-dimensional graphical representation of the equilibrium payoffs induced by the bilateral bargaining procedure. Figure 12 displays the payoffs that A and C get against those of B and D. In Subgame 1, A and C get the largest joint payoff while B and D get the smallest joint payoff within the core. The opposite happens in Subgame 2. The expected payoff lies halfway on the segment between the joint payoffs corresponding to each subgame. Allocations that yield joint payoffs outside this segment are not in the core.

The individual payoffs to A and C are shown in the first panel of Figure 13. Payoffs outside the heavy square lie outside the core. In Subgame 1 the payoffs to A and C are given by the upper-right corner of the square. Conversely, their payoffs in Subgame 2 are given by the lower-left corner of the box. The expected payoffs to A and C lie at the center of the square. Similarly, the second panel of Figure 13 shows the payoffs to B and D. And again, every payoff profile Π in the core must have (Π_D, Π_B) inside the heavy square. The upper-right corner of this box represents D and B's payoffs in Subgame 2, when they get to make the take-it-or-leave-it offers. Their payoffs in Subgame 1, when A and C get to make take-it-or-leave-it offers, are on the lower-left corner of the box. Their expected payoffs lie in the middle of the square.

Next consider the case $V_i < V_k < V_j < V_i + V_j - V_0$. For this case it can be shown that any payoff profile $\Pi = (\Pi_A, \Pi_B, \Pi_C, \Pi_D)$ in the core must satisfy: $V_0 \leq \Pi_A \leq 2V_i - V_0$, $V_0 \leq \Pi_B \leq 2V_i - V_0$, $V_0 + 2(V_k - V_i) \leq \Pi_C \leq 2V_j - V_0$, $V_0 \leq \Pi_D \leq 2(V_i + V_j - V_k) - V_0$. Figure 14 displays the payoffs that A and C get against those of B and D. In Subgame 1 A and C get the biggest joint payoff while B and D get the smallest joint payoff of any core allocation. The opposite happens in Subgame 2. The expected payoff lies halfway on the segment between the joint payoffs corresponding to each subgame. Allocations that yield joint payoffs outside this segment are not in the core. The individual payoffs to A and C are shown in the first panel of Figure 15. Payoffs outside the heavy rectangle lie outside the core. In Subgame 1 the payoffs to A and C are given by the upper-right corner of the rectangle. Their payoffs in Subgame 2 are given by the lower-left corner of the rectangle. The expected payoffs to A and B lie at the center of the rectangle. The second panel of Figure 15 shows the payoffs to B and D.

Finally, consider the case $V_k < V_i < V_j < V_i + V_j - V_0$. For this case it can be shown that any payoff profile $\Pi = (\Pi_A, \Pi_B, \Pi_C, \Pi_D)$ in the core must satisfy $V_0 \leq \Pi_A \leq 2V_i - V_0$, $V_0 \leq \Pi_B \leq 2V_i - V_0$, $V_0 \leq \Pi_C \leq 2V_j - V_0$, and $V_0 \leq \Pi_D \leq 2V_j - V_0$. Figure 16 displays the payoffs that A and C get against those of B and D. In Subgame 1 A and C get the biggest joint payoff while B and D get the smallest joint payoff of any core allocation. The opposite happens in Subgame 2. The expected joint payoff lies halfway on the segment between the joint payoffs corresponding to each subgame. Allocations that yield joint payoffs outside this segment are not in the core. The individual payoffs to A and C are shown in the first panel of Figure 17. Payoffs outside the heavy rectangle lie outside the core. In Subgame 1 the payoffs to A and C are given by the upper-right corner of the rectangle. Conversely, their payoffs in Subgame 2 are given by the lower-left corner of the rectangle. The expected payoffs to A and B lie at the center of the rectangle. Similarly, the second panel of Figure 17 shows the payoffs to B and D.

The model with firing taxes.

We now analyze the bargaining procedure in the presence of firing taxes (Section 5.2). Start with the single-breach situation of Figure 2. The bargaining procedure is:

Subgame 1. With probability a half, B makes a take-it-or-leave-it offer specifying continuation payoffs as well as a proposal to engage in joint production to either A or C. If B was to offer continued joint production to A, he would offer A her minimum acceptable payoff, $X_{BA}^k = V_0 + S_{i0}^k$. Agent A would accept the offer and B's payoff from continued production with A would then be $2V_i - V_0 - S_{i0}^k$. Alternatively, if B was to offer joint production to C he would offer her $X_{BC}^k = V_0$, her minimum acceptable continuation value. C would accept this offer and B's payoff after paying the firing compensation to A would be $2V_k - V_0 - T_{i0}^k$. If $V_k > V_i + \frac{1}{2}\left(T_{i0}^k - S_{i0}^k\right)$ then B will choose to leave A and form a new match with C. The payoffs to A, B and C will be $V_0 + S_{i0}^k$, $2V_k - V_0 - T_{i0}^k$, and V_0 respectively. Alternatively, if $V_k \leq V_i + \frac{1}{2}\left(T_{i0}^k - S_{i0}^k\right)$, then B will offer continued production to A and the payoffs to A, B and C will be $V_0 + S_{i0}^k$, $2V_i - V_0 - S_{i0}^k$, and V_0.

Subgame 2. With probability another half, A and C simultaneously make offers to B. Since A's outside option is now $V_0 + S_{i0}^k$, she is willing to offer B no more than $2V_i - V_0 - S_{i0}^k$. On the other hand, the maximum C is willing to offer B is $2V_k - V_0$. Therefore A offers B a continuation payoff $X_{AB}^i = \max\left[\min\left(2V_i - V_0 - S_{i0}^k, 2V_k - V_0 - T_{i0}^k + \varepsilon\right), V_0 + S_{i0}^k\right]$ and C's offer is for B's continuation payoff to be $X_{CB}^k = \min\left(2V_k - V_0 - T_{i0}^k, 2V_i - V_0 - S_{i0}^k + \varepsilon\right)$ where ε is an arbitrarily small positive number.[27] If $V_k > V_i + \frac{1}{2}\left(T_{i0}^k - S_{i0}^k\right)$ then B forms a new match with C and the payoffs to A, B and C are $V_0 + S_{i0}^k$, $2V_i - V_0 - S_{i0}^k$, and $2V_k - \left(2V_i - V_0 - S_{i0}^k + T_{i0}^k\right)$ respectively. Conversely, if $V_k \leq V_i + \frac{1}{2}\left(T_{i0}^k - S_{i0}^k\right)$ then B stays matched to A and the payoffs to A, B and C are $2V_i - \left(2V_k - V_0 - T_{i0}^k\right)$, $2V_k - V_0 - T_{i0}^k$, and V_0 respectively.

In both subgames B leaves A for sure if and only if $V_k > V_i + \frac{1}{2}\left(T_{i0}^k - S_{i0}^k\right)$; or equivalently,

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[27] The compensation T_{i0}^k appears subtracting from the second argument of the "min" in X_{AB}^k and from the first argument of the "min" in X_{CB}^k because when C transfers $2V_k - V_0$ to B, if B matches with C he only gets $2V_k - V_0 - T_{ij}^k$ after paying the firing tax. Since $S_{ij}^k \leq V_i - V_0$, agent A always wants to preserve her match with B; the "max" in X_{CB}^k ensures that A offers B a continuation payoff at least equal to her outside option , $V_0 + S_{i0}^k$, even if C's offer to B is $2V_k - V_0 - T_{i0}^k < V_0 + S_{i0}^k$.

if $2V_k - T_{i0}^k + V_0 + S_{i0}^k > 2V_i + V_0$, i.e. if and only if the total surplus of A, B and C from forming the new match after paying the net tax to the government (the left-hand side) exceeds the total surplus associated with maintaining the existing match. In this case the expected capital gains are

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \end{bmatrix} = \begin{bmatrix} -\left(V_i - V_0 - S_{i0}^k\right) \\ V_k - V_0 - \frac{1}{2}\left(T_{i0}^k + S_{i0}^k\right) \\ V_k - V_i - \frac{1}{2}\left(T_{i0}^k - S_{i0}^k\right) \end{bmatrix}.$$

If $V_0 + \frac{1}{2}\left(T_{i0}^k + S_{i0}^k\right) \leq V_k \leq V_i + \frac{1}{2}\left(T_{i0}^k - S_{i0}^k\right)$, then A and B preserve their match and the expected capital gains are

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \end{bmatrix} = \begin{bmatrix} -\left[V_k - V_0 - \frac{1}{2}\left(T_{i0}^k + S_{i0}^k\right)\right] \\ V_k - V_0 - \frac{1}{2}\left(T_{i0}^k + S_{i0}^k\right) \\ 0 \end{bmatrix}.$$

If $V_k < V_0 + \frac{1}{2}\left(T_{i0}^k + S_{i0}^k\right)$, then B remains matched to A and is unable to extract a positive expected side payment from her: all agents' continuation payoffs remain unchanged and nobody experiences capital gains or losses.

Next, consider the double-breach situation illustrated in Figure 3.

Subgame 1. With probability a half, A and C simultaneously make offers to B. C also makes a take-it-or-leave-it offer to his existing partner D, and this offer is contingent on his offer to B being rejected. C makes the smallest acceptable offer to D, namely $V_0 + S_{ji}^k$. The resulting payoff to C from continuing to match with D is $2V_j - V_0 - S_{ji}^k$, which constitutes the opportunity cost for C to form a new match. Thus the maximum utility C is willing to give up to attract B (i.e. the utility transfer to B that would make C just indifferent between staying with D and forming a new match with B) is $2V_k - T_{ji}^k - (2V_j - V_0 - S_{ji}^k)$. This transfer would guarantee B a continuation payoff equal to $2V_k - (2V_j - V_0 + T_{ij}^k + T_{ji}^k - S_{ji}^k)$ (net of his tax liability T_{ij}^k for separating from A). If B "fires" A, then A's continuation payoff is $V_0 + S_{ij}^k$. Thus the maximum A is willing to offer B is $2V_i - V_0 - S_{ij}^k$. Since this valuation is nonnegative (recall that $S_{ij}^k \leq V_i - V_0$), A will want to make sure that B finds her offer acceptable, and for this she must ensure that B's payoff is at least as large as V_0. Therefore, A offers B a continuation payoff $X_{AB}^i = Max\{V_0 + S_{ij}^k, Min[2V_i - V_0 - S_{ij}^k, 2V_k - 2V_j + V_0 + S_{ji}^k - T_{ji}^k - T_{ij}^k + \varepsilon]\}$ and

C offers B's payoff to be $X^k_{CB} = Min[2V_k - 2V_j + V_0 + S^k_{ji} - T^k_{ji} - T^k_{ij}, 2V_i - V_0 - S^k_{ij} + \varepsilon]$ for an arbitrarily small positive ε. Then, B will accept C's offer to form the new match for sure if and only if $V_k + V_0 - V_i - V_j > \frac{1}{2}\left(T^k_{ij} + T^k_{ji} - S^k_{ij} - S^k_{ji}\right)$.

Subgame 2. With probability another half, B and D simultaneously make offers to C. B also makes an offer to his current partner A, and this offer is contingent on his offer to C being rejected. The analysis of this subgame parallels that of subgame 1 so we omit it.

In the two possible sequences of bargaining (subgame 1 and subgame 2) B and C abandon their old partners to form a new match for sure if and only if the sum of the value of the new match and the unmatched after paying the net tax to the government exceeds the sum of two existing matches; i.e. if and only if $2V_k + 2V_0 - \left(T^k_{ij} + T^k_{ji} - S^k_{ij} - S^k_{ji}\right) > 2V_i + 2V_j$. The equilibrium expected gains are:

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} -(V_i - V_0 - S^k_{ij}) \\ V_k - V_j - \frac{1}{2}\left(T^k_{ij} + S^k_{ij} + T^k_{ji} - S^k_{ji}\right) \\ V_k - V_i - \frac{1}{2}\left(T^k_{ji} + S^k_{ji} + T^k_{ij} - S^k_{ij}\right) \\ -(V_j - V_0 - S^k_{ji}) \end{bmatrix}$$

if $V_i + V_j - V_0 + \frac{1}{2}\left(T^k_{ij} + T^k_{ji} - S^k_{ij} - S^k_{ji}\right) < V_k$; and

$$\begin{bmatrix} \Gamma_A \\ \Gamma_B \\ \Gamma_C \\ \Gamma_D \end{bmatrix} = \begin{bmatrix} -\max\left[V_k - V_j - \frac{1}{2}\left(T^k_{ij} + S^k_{ij} + T^k_{ji} - S^k_{ji}\right), 0\right] \\ \max\left[V_k - V_j - \frac{1}{2}\left(T^k_{ij} + S^k_{ij} + T^k_{ji} - S^k_{ji}\right), 0\right] \\ \max\left[V_k - V_i - \frac{1}{2}\left(T^k_{ji} + S^k_{ji} + T^k_{ij} - S^k_{ij}\right), 0\right] \\ -\max\left[V_k - V_i - \frac{1}{2}\left(T^k_{ji} + S^k_{ji} + T^k_{ij} - S^k_{ij}\right), 0\right] \end{bmatrix}$$

if $V_k \leq V_i + V_j - V_0 + \frac{1}{2}\left(T^k_{ij} + T^k_{ji} - S^k_{ij} - S^k_{ji}\right)$. If $V_k < V_j + \frac{1}{2}\left(T^k_{ij} + S^k_{ij} + T^k_{ji} - S^k_{ji}\right)$, then B remains matched to A and is unable to use his meeting with C to extract a side payment from A. Similarly, C is unable to extract a side payment from D if $V_k < V_i + \frac{1}{2}\left(T^k_{ji} + S^k_{ji} + T^k_{ij} - S^k_{ij}\right)$.

It follows from the previous analysis that firing taxes will in general alter the match formation and destruction decisions. Summarizing, in the single-breach situation of Figure 2, B will destroy his match with A to form a new one with C if and only if $V_k > V_i + \frac{1}{2}\left(T^k_{i0} - S^k_{i0}\right)$. And in the double-breach situation of Figure 3 B and C leave their current partners if and only if

$V_k + V_0 > V_i + V_j + \frac{1}{2}\left(T_{ij}^k + T_{ji}^k - S_{ij}^k - S_{ji}^k\right)$. Using the equilibrium break up rules and focusing on a symmetric equilibrium $\widehat{\phi}_{ij}^k = \phi_{ij}^k$, the Bellman equations are as reported in the main text.

References

[1] Albæk, Karsten, and Bent E. Sørensen. "Worker Flows and Job Flows in Danish Manufacturing, 1980-91." *The Economic Journal* 105(451) (November 1998): 1750-1771.

[2] Anderson, Patricia, and Bruce D. Meyer. "The Extent and Consequences of Job Turnover." Brookings Papers on Economic Activity. Microeconomics, Vol. 1994 (1994): 177-248.

[3] Bertola, Giuseppe, and Richard Rogerson. "Institutions and Labor Reallocation." *European Economic Review* 41 (1997): 1147-1171.

[4] Blanchard, Olivier, and Pedro Portugal. "What Hides Behind an Unemployment Rate: Comparing Portuguese and U.S. Labor Markets". *American Economic Review* 91(1) (March 2001): 187-207.

[5] Bontemps, Christian, Jean-Marc Robin, and Gerard J. van den Berg. "An Empirical Equilibrium Job Search Model with Search On the Job and Heterogeneous Workers and Firms." *International Economic Review* 40(4) (November 1999): 1039-1074.

[6] Bontemps, Christian, Jean-Marc Robin, and Gerard J. van den Berg. "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation." *International Economic Review* 41(2) (May 2000): 305-358.

[7] Broersma, Lourens, den Butter, Frank A. G. and Udo Kock. "A National Accounting System for Worker Flows." *Economics Letters* 67 (2000): 331-336.

[8] Burdett, Kenneth. "A Theory of Employee Job Search and Quit Rates." *American Economic Review* 68(1) (March 1978): 212-220.

[9] Burdett, Kenneth, and Melvyn Coles. "Marriage and Class." *Quarterly Journal of Economics* 112(1) (February 1997): 141-168.

[10] Burdett, Kenneth, and Melvyn Coles. “Equilibrium Wage Tenure Contracts.” *Econometrica* 71(5): 1377-1404.

[11] Burdett, Kenneth, Ryoichi Imai, and Randall Wright. “Unstable Relationships.” *Frontiers of Macroeconomics*: Vol. 1: No. 1, Article 1, 2004.

[12] Burdett, Kenneth, Ricardo Lagos, and Randall Wright. “An On-the-Job Search Model of Unemployment, Inequality, and Crime.” *International Economic Review*, forthcoming 2004.

[13] Burdett, Kenneth, and Dale T. Mortensen. “Wage Differentials, Employer Size and Unemployment,” *International Economic Review* 39 (1998): 257-73.

[14] Burgess, Simon, Julia Lane, and David Stevens. “Job Flows, Worker Flows, and Churning.” *Journal of Labor Economics* 18(3) (July 2000): 473-502.

[15] Coles, Melvyn. “Equilibrium Wage Dispersion, Firm Size, and Growth.” *Review of Economic Dynamics* 4(1) (January 2001): 159-187.

[16] Cornelius, Tracy. “A Search Model of Marriage and Divorce.” *Review of Economic Dynamics* 6(1) (January 2003): 135-155.

[17] Davis, Steven J., and John Haltiwanger. “Gross Job Creation, Gross Job Destruction, and Employment Reallocation.” *Quarterly Journal of Economics* 107(3) (August 1992): 819-863.

[18] Davis, Steven J., and John Haltiwanger. “Measuring Gross Worker and Job Flows.” Labor Statistics Measurement Issues, edited by John Haltiwanger, Marilyn Manser and Robert Topel. University of Chicago Press. Chicago, 1999, pp. 77-122.

[19] Davis, Steven J., John Haltiwanger and Scott Schuh. *Job Creation and Destruction*. MIT Press, 1996.

[20] Dey, Matthew S., and Christopher J. Flinn. “An Equilibrium Model of Health Insurance Provision and Wage Determination.” New York University and University of Chicago mimeo, 2000.

[21] Diamond, Peter A., and Eric S. Maskin. “An Equilibrium Analysis of Search and Breach of Contract I: Steady States.” *Bell Journal of Economics* 10(1) (September 1979): 282-316.

[22] Diamond, Peter A., and Eric S. Maskin. “An Equilibrium Analysis of Search and Breach of Contract II: A Non-Steady State Example.” *Journal of Economic Theory* 25(2) (October 1981): 165-195.

[23] Diamond, Peter A., and Eric S. Maskin. “Externalities and Efficiency in a Model of Stochastic Job Matching.” MIT mimeo, November 1981.

[24] Eshel, Ilan, and Avner Shaked. “Partnership.” *Journal of Theoretical Biology*, 208 (2001): 457-474.

[25] Fallick, Bruce C., and Charles A. Fleischman. “The Importance of Employer-to-Employer Flows in the U.S. Labor Market.” Federal Reserve Board, mimeo, April 2001.

[26] Garcia-Serrano, Carlos and Juan Francisco Jimeno. “Labor Reallocation, Job Tenure, Labor Flows and Labor Market Institutions: Evidence from Spain.” FEDEA working paper, 1998.

[27] Hamermesh, Daniel S., Wolter H. J. Hassink and Jan C. van Ours. “Job Turnover and Labor Turnover: A Taxonomy of Employment Dynamics.” *Annales D’Économie et de Statistique* 41/42 (1996): 21-40.

[28] Lazear, Edward. P. “Job Security Provisions and Employment." *Quarterly Journal of Economics* 105(3) (August 1990): 699-726.

[29] Manning, Alan. *Monopsony in Motion: Imperfect Competition in Labor Markets.* Princeton University Press, 2003.

[30] Mortensen, Dale T. "Specific Capital and Labor Turnover." *Bell Journal of Economics* 9(2) (Autumn 1978): 572-586.

[31] Mortensen, Dale T. "Property Rights and Efficiency in Mating, Racing, and Related Games." *American Economic Review* 72(5) (1982): 968-79.

[32] Mortensen, Dale T. "The Matching Process as a Non-Cooperative/Bargaining Game." In *The Economics of Information and Uncertainty*, ed. J. J. McCall. Chicago: University of Chicago Press, 1982.

[33] Mortensen, Dale T. *Wage Dispersion: Why Are Similar Workers Paid Differently?* MIT Press, 2003

[34] Mortensen, Dale T., and Christopher A. Pissarides. "Job Creation and Destruction in the Theory of Unemployment." *Review of Economic Studies* 61(3): 397-415.

[35] Nagypál, Éva. "Worker Reallocation over the Business Cycle: The Importance of Job-to-Job Transitions." Northwestern University mimeo, November 2003.

[36] Pissarides, Christopher A. "Search Unemployment with On-the-job Search." *Review of Economic Studies* 61(3). (July 1994): 457-475.

[37] Pissarides, Christopher A. *Equilibrium Unemployment Theory.* Cambridge: MIT Press, 2000 (second edition).

[38] Postel-Vinay, Fabien, and Jean-Marc Robin. "To Match Or Not To Match? Optimal Wage Policy with Endogenous Worker Search Intensity." *Review of Economic Dynamics*, forthcoming 2003.

[39] Postel-Vinay, Fabien, and Jean-Marc Robin. "Wage Dispersion with Worker and Employer Heterogeneity." *Econometrica* 70(6) (2002): 2295-350.

[40] Postel-Vinay, Fabien, and Jean-Marc Robin. "The Distribution of Earnings in an Equilibrium Search Model with State-Dependent Offers and Counter-Offers." *International Economic Review* 43(4) (2002): 989-1016.

[41] Shimer, Robert. "On-the-Job Search, Bargaining, and Wage Dispersion." University of Chicago mimeo, 2004.

[42] Shimer, Robert, and Lones Smith. "Assortative Matching and Search." *Econometrica* 68(2) (March 2000): 343-369.

[43] Shimer, Robert, and Lones Smith. "Nonstationary Search." University of Chicago and University of Michigan mimeo, 2001.

[44] Stewart, Jay. "Recent Trends in Job Stability and Job Security: Evidence from the March CPS." Bureau of Labor Statistics mimeo, February 2002.

[45] Van den Berg, Gerard J., and Geert Ridder. "An Empirical Equilibrium Search Model of the Labor Market." *Econometrica* 66(5) (September 1998): 1183-1221.

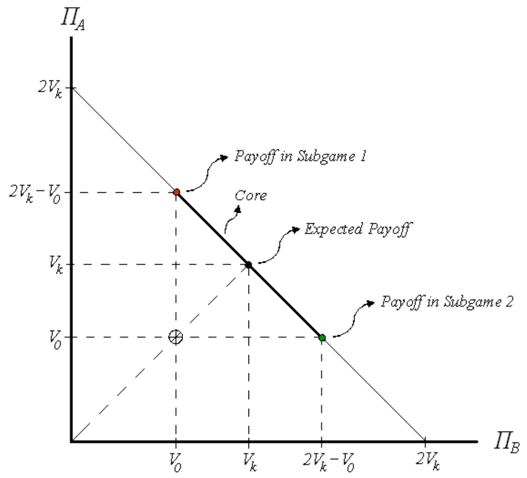
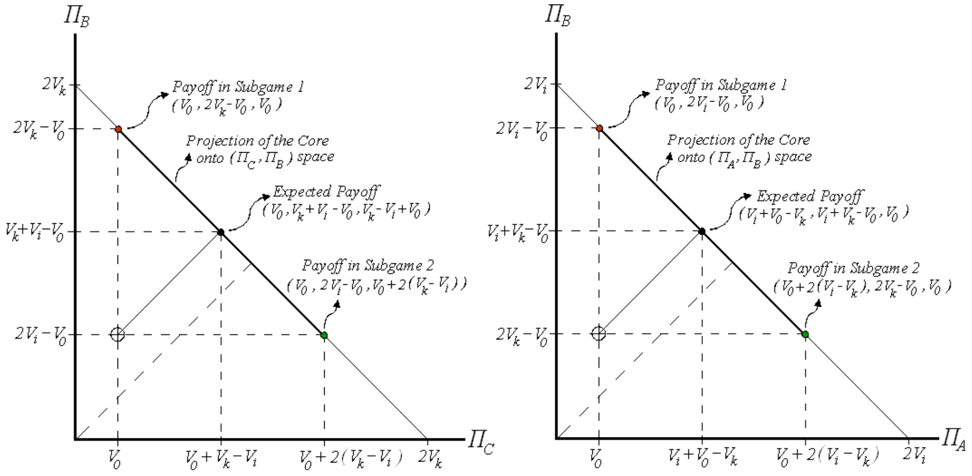


Figure 9: Core payoffs for a meeting involving two agents.



(i). $V_k > V_i$

(ii). $V_i > V_k$

Figure 10: Core payoffs for a meeting involving three agents.

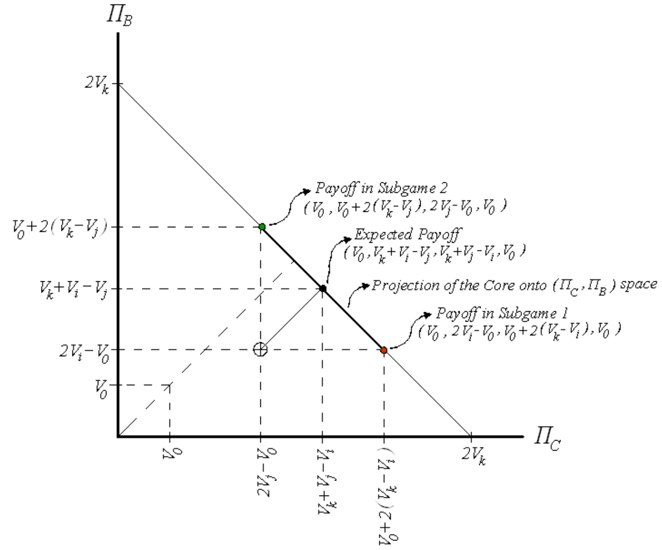


Figure 11: Core payoffs for a meeting involving four agents with $V_i < V_j$ and $V_i + V_j - V_0 < V_k$.

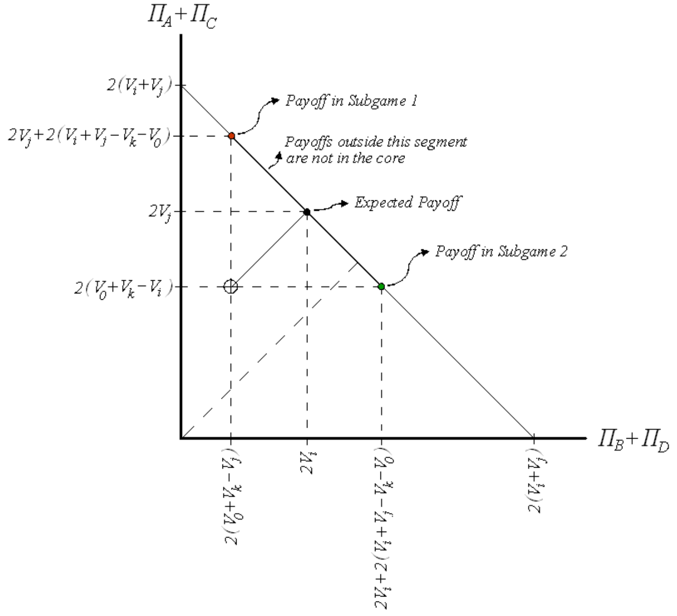


Figure 12: Joint payoffs for a meeting involving four agents with $V_i < V_j < V_k < V_i + V_j - V_0$.

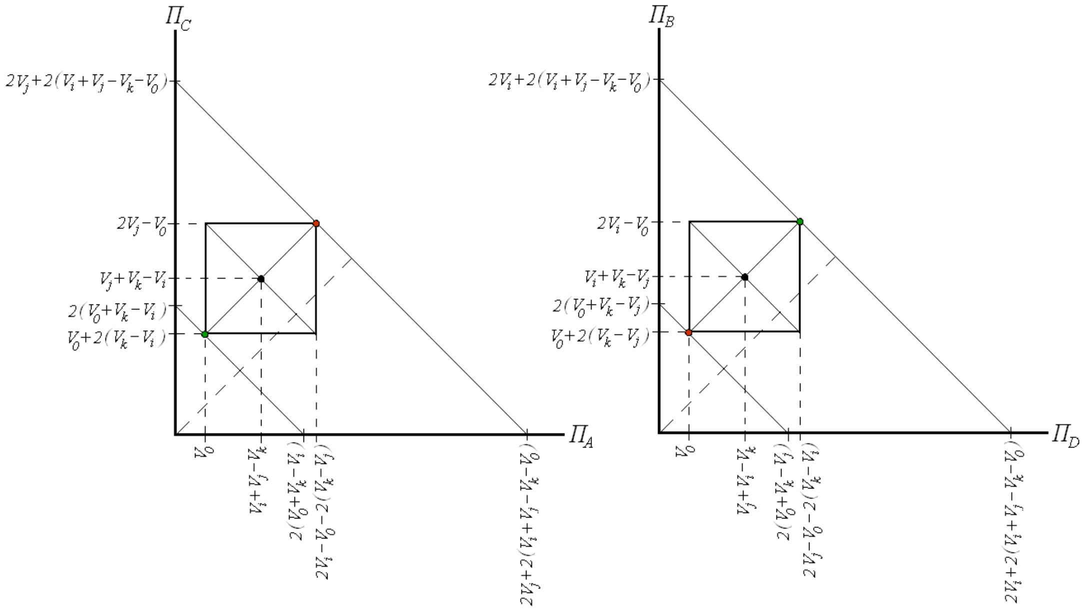


Figure 13: Individual payoffs for a meeting with four agents and $V_i < V_j < V_k < V_i + V_j - V_0$.

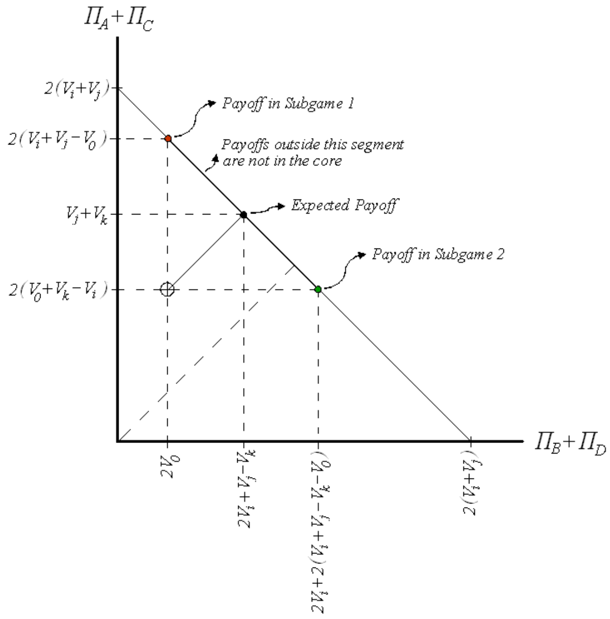


Figure 14: Joint payoffs for a meeting involving four agents with $V_i < V_k < V_j < V_i + V_j - V_0$.

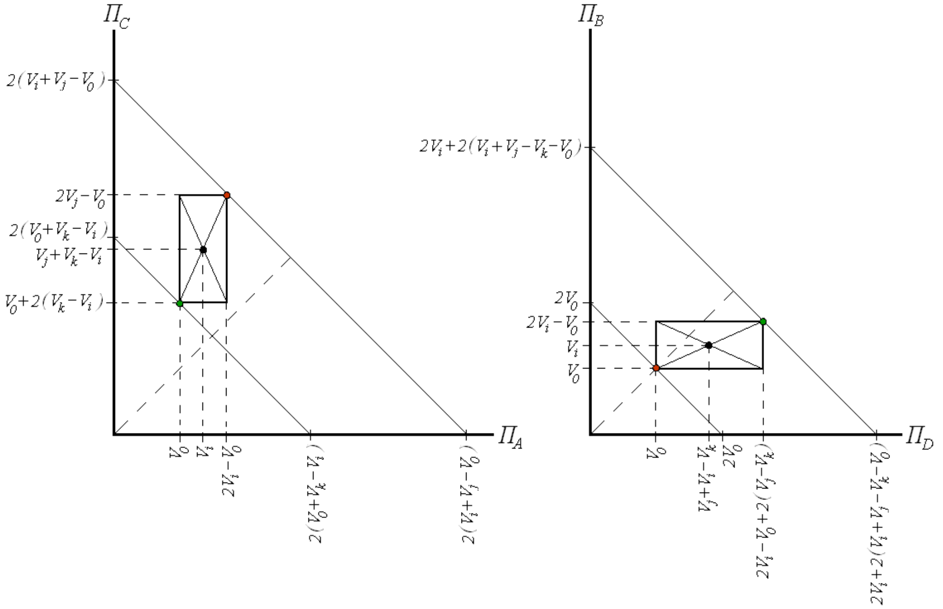


Figure 15: Individual payoffs for a meeting with four agents and $V_i < V_k < V_j < V_i + V_j - V_0$.

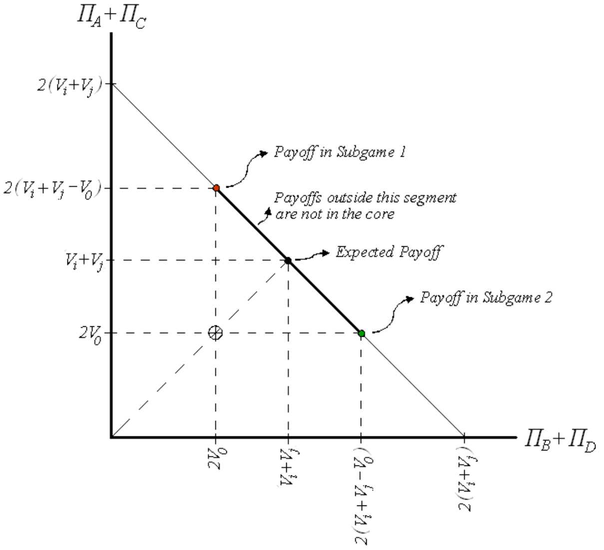


Figure 16: Joint payoffs for a meeting involving four agents with $V_k < V_i < V_j < V_i + V_j - V_0$.

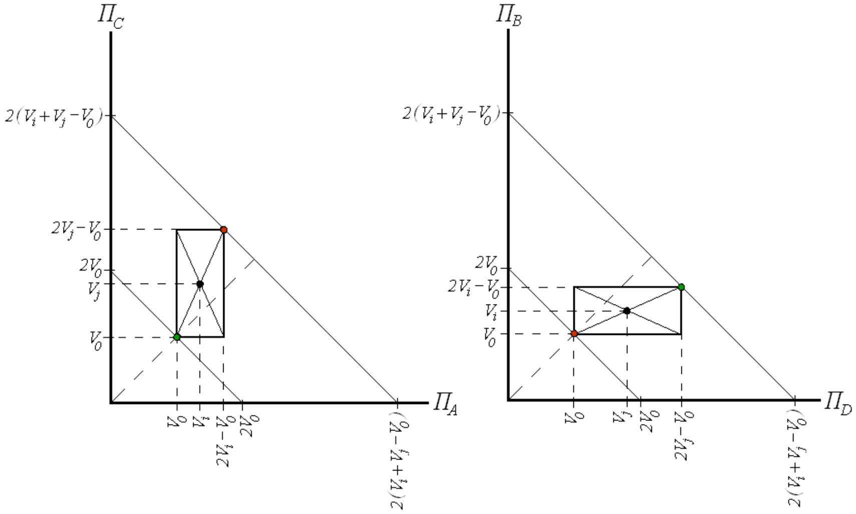


Figure 17: Individual payoffs for a meeting with four agents and $V_k < V_i < V_j < V_i + V_j - V_0$.